Widest Paths and Global Propagation in Bounded Value Iteration for Stochastic Games

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Summary

We introduce a novel algorithm of
Bounded Value Iteration (BVI) for Stochastic Games.

<u>What is BVI?</u>

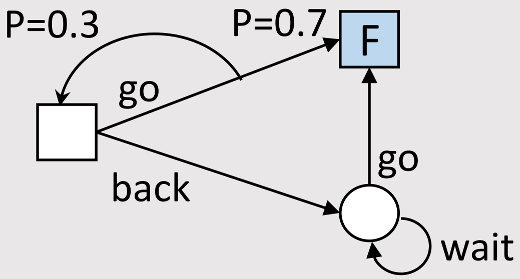
- Approximation technique for reachability
- Approximation with precision guarantee
  - "Compute reachability prob. with 0.01% error range"

<u>Our contribution: faster algorithm</u>

- Existing algorithm [Kelmendi+, CAV'18] requires end component computation
- We omit it by doing global propagation

Our model: Stochastic Game (SG)

- A probabilistic system with controller and adversary
- Discrete time, finite states / actions
- Reachability objective



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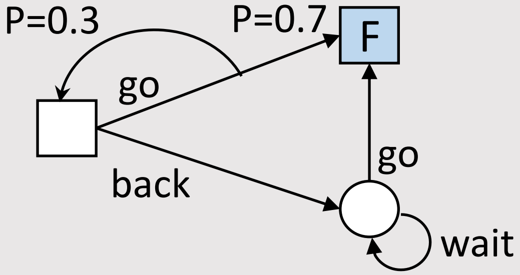
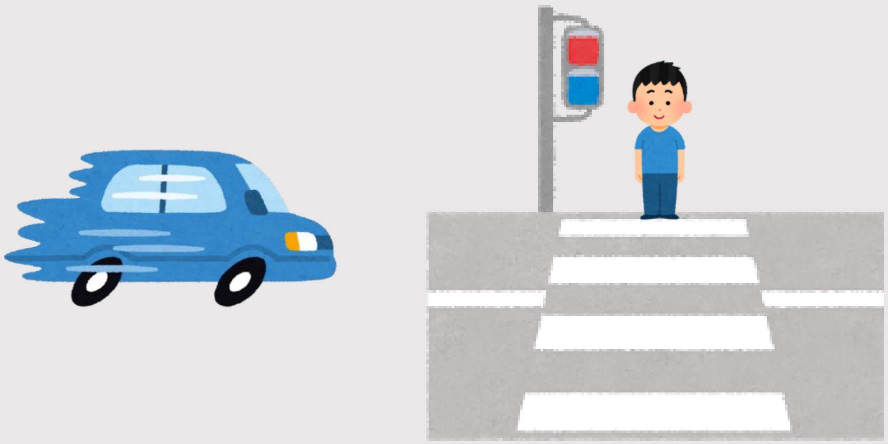
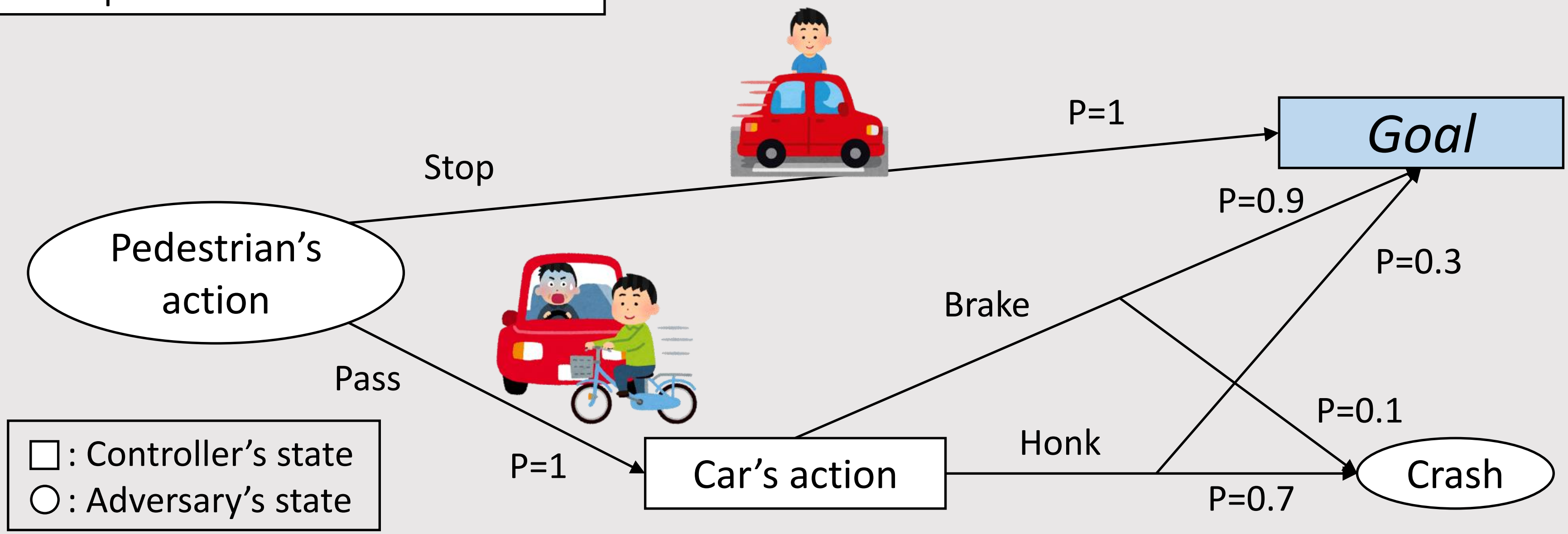


Example: car vs. pedestrian

- The car (controller) would like to pass the crossroad without hitting the pedestrian (adversary)



Car vs. pedestrian in Stochastic Game



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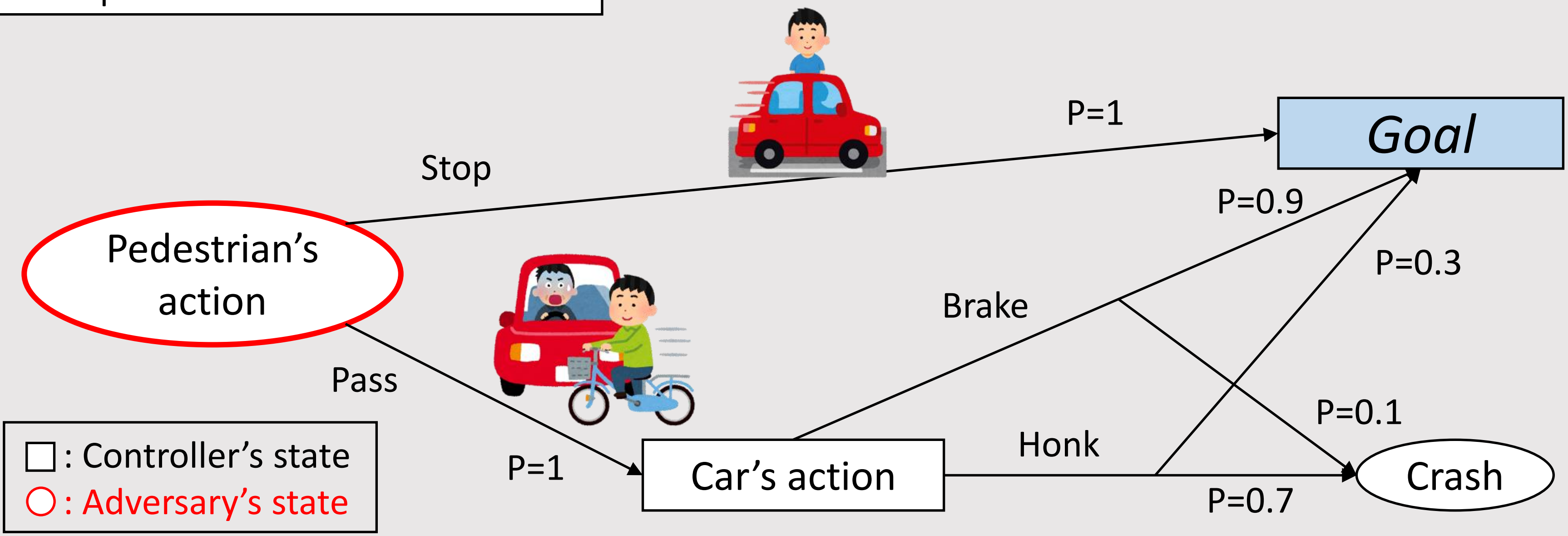

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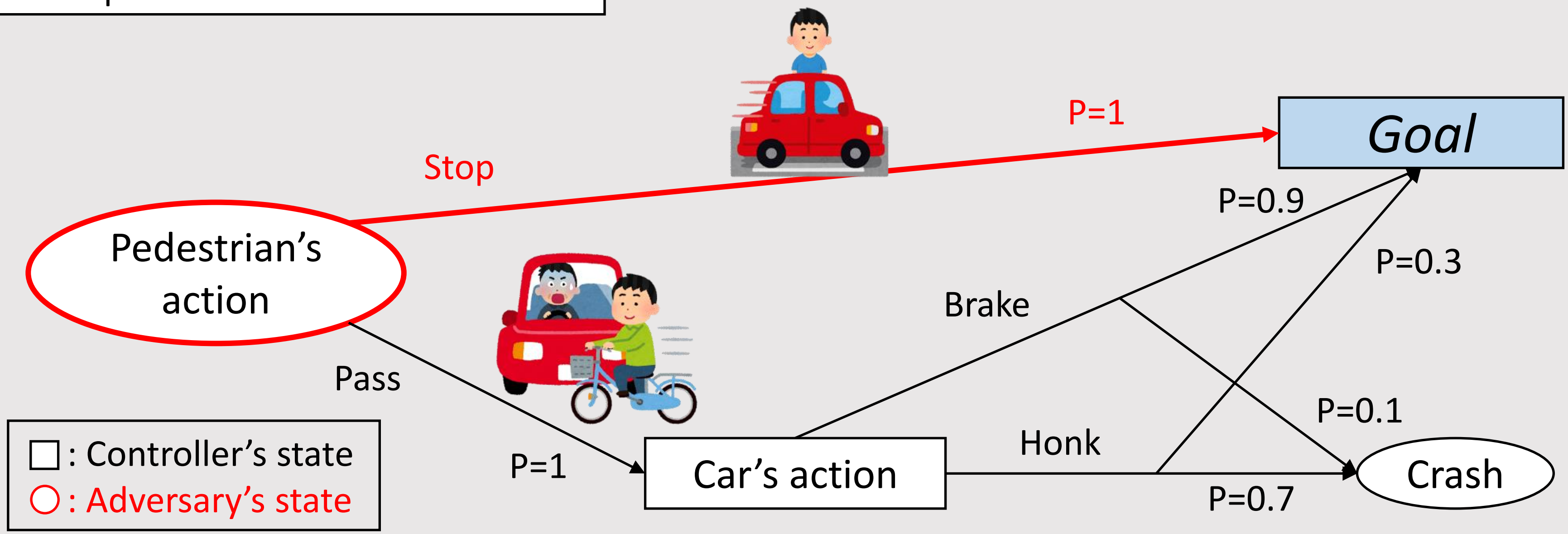

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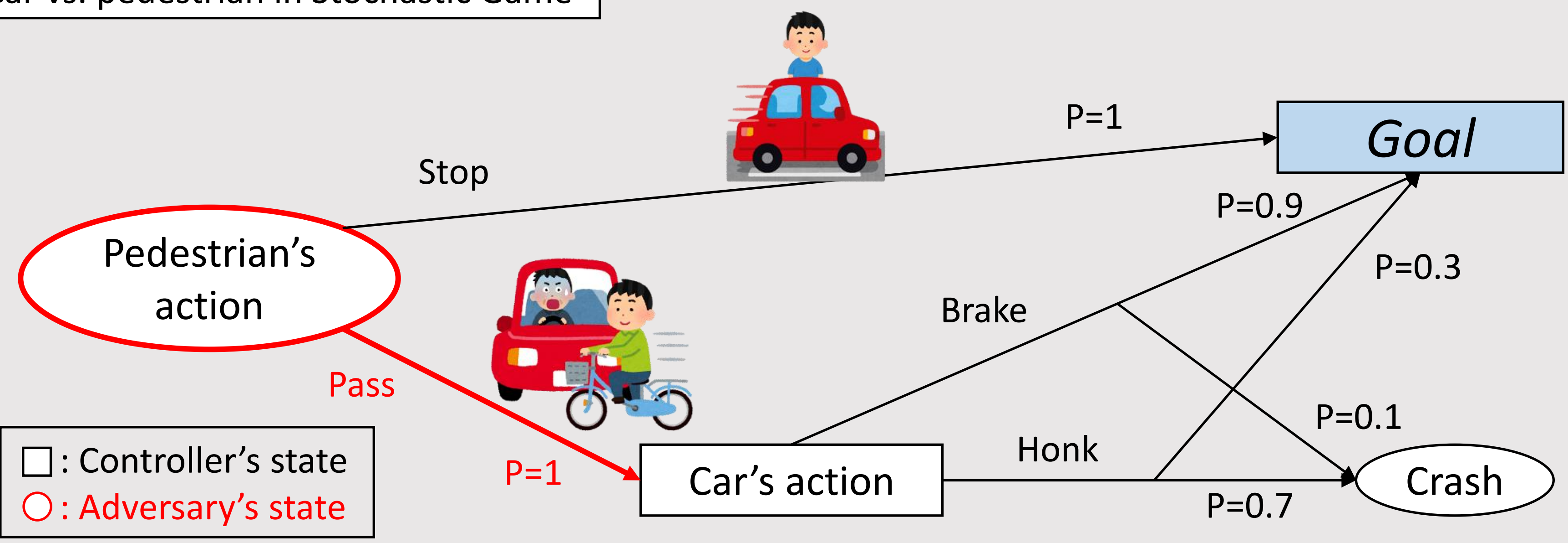

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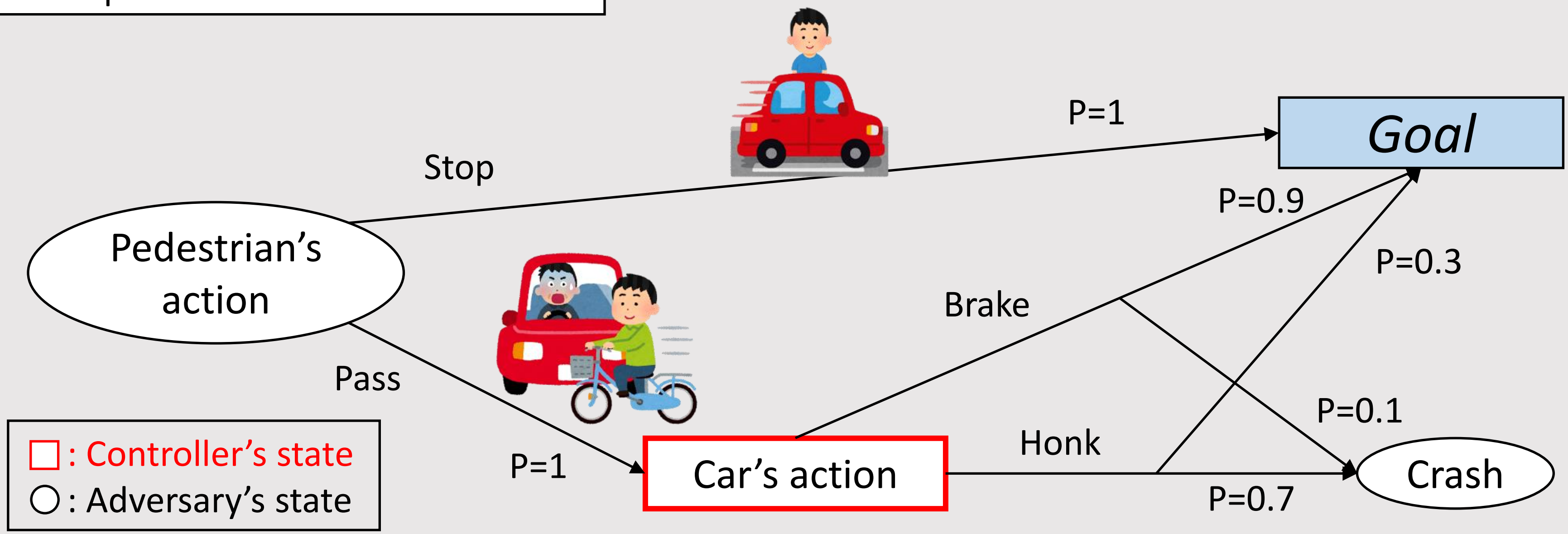

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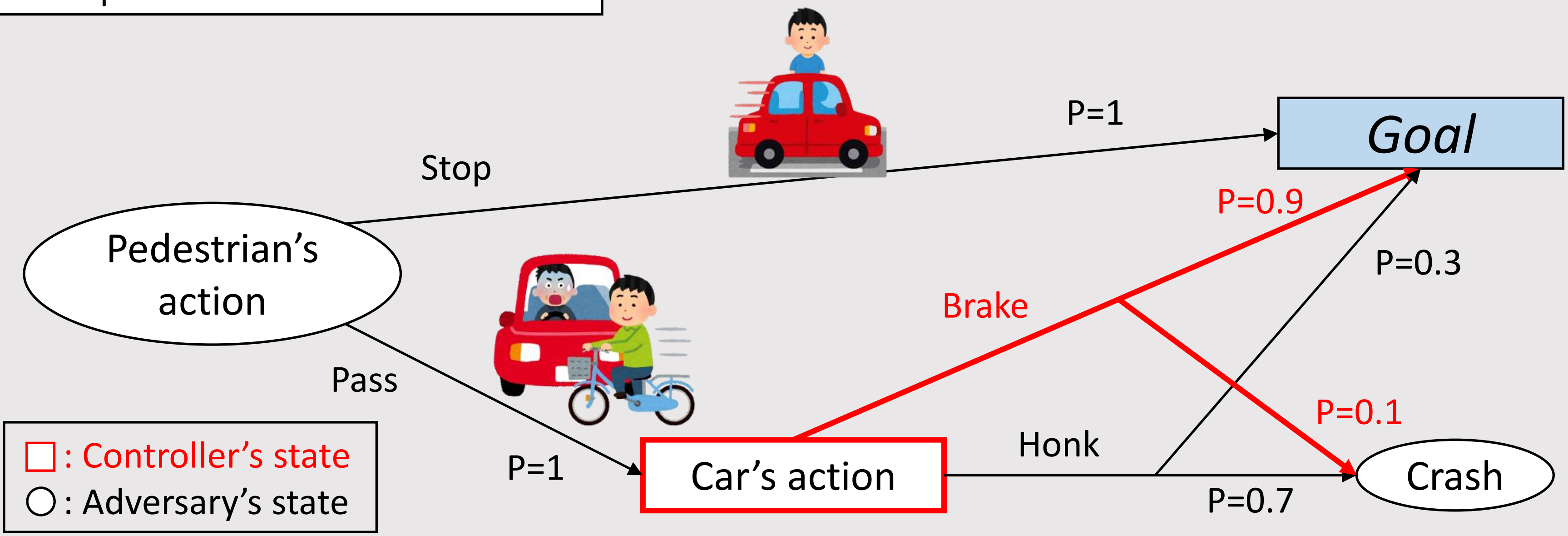

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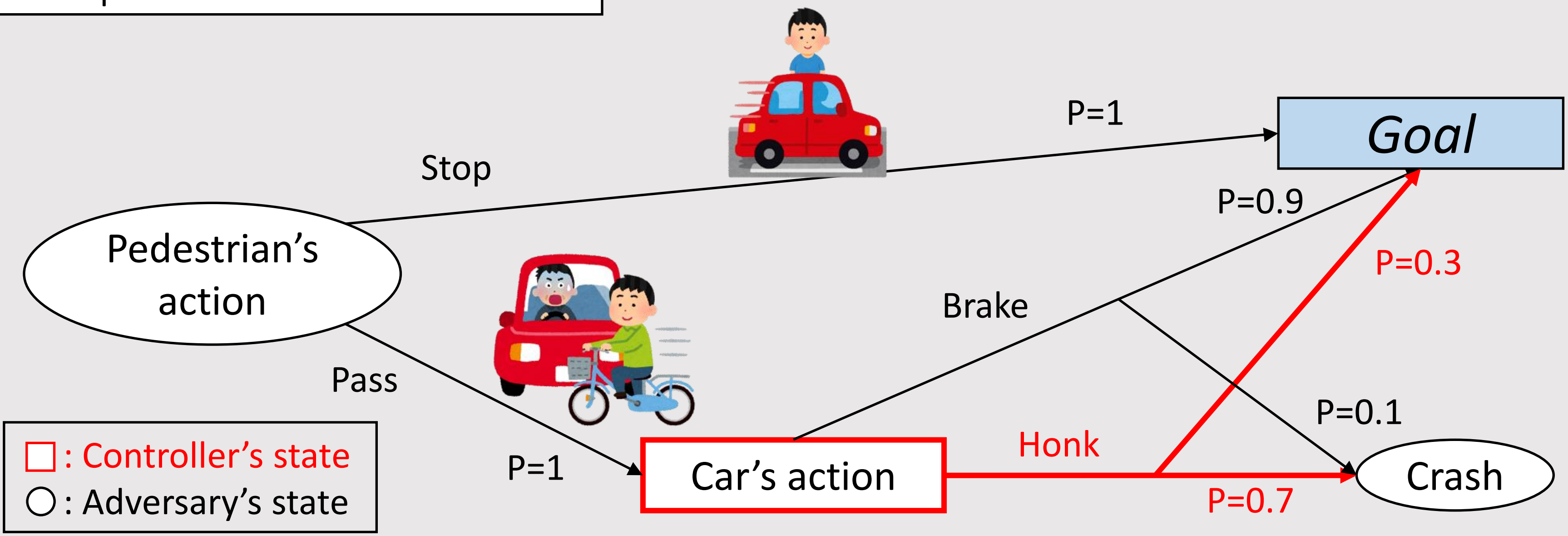

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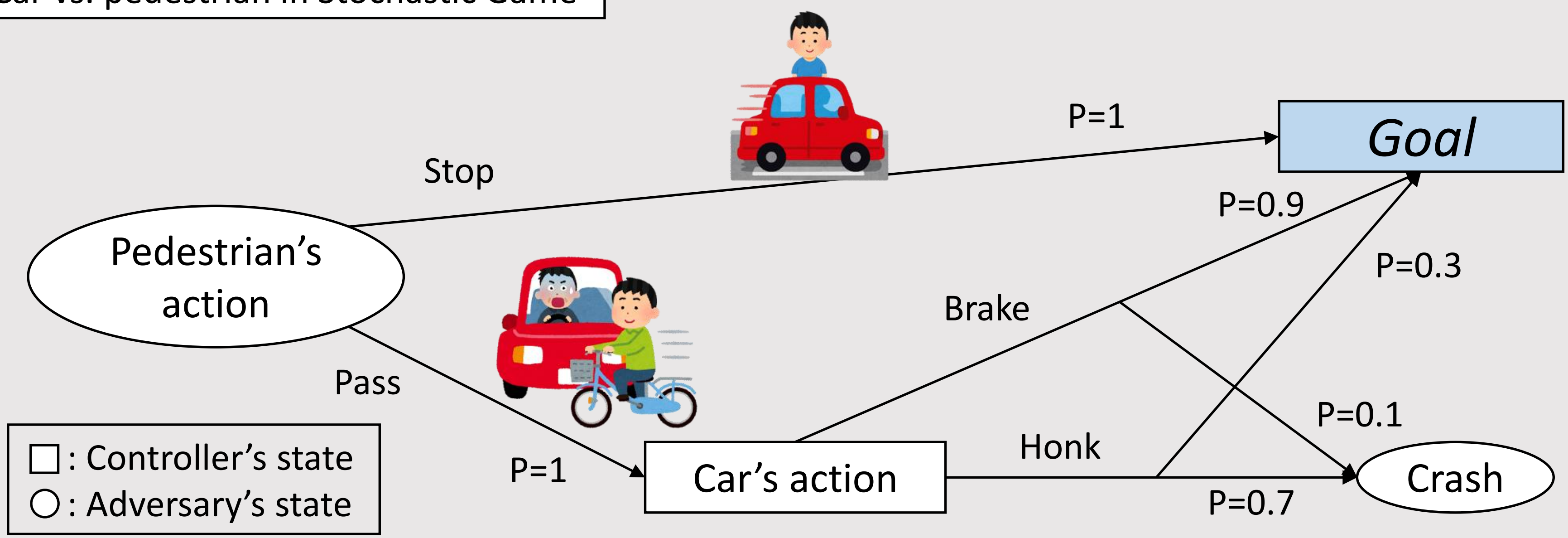

- A (pure positional) strategy of player X … $\sigma: (\mathrm{X}'\mathrm{s\ states}) \to (\mathrm{actions})$

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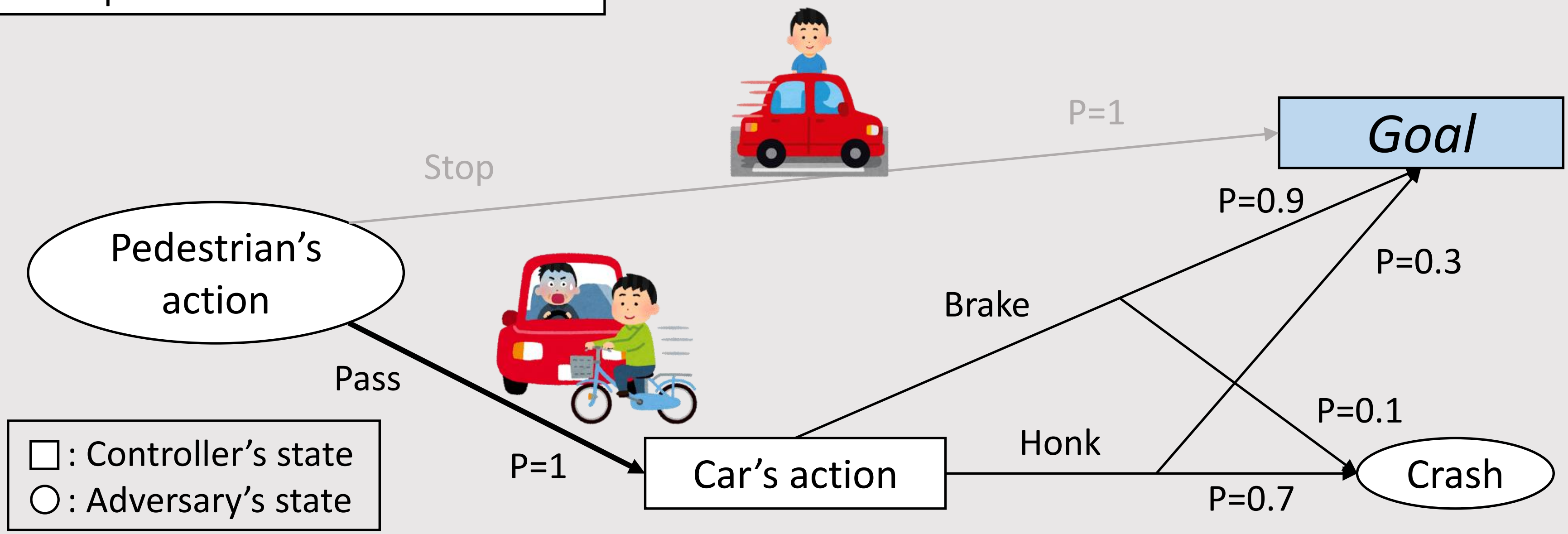

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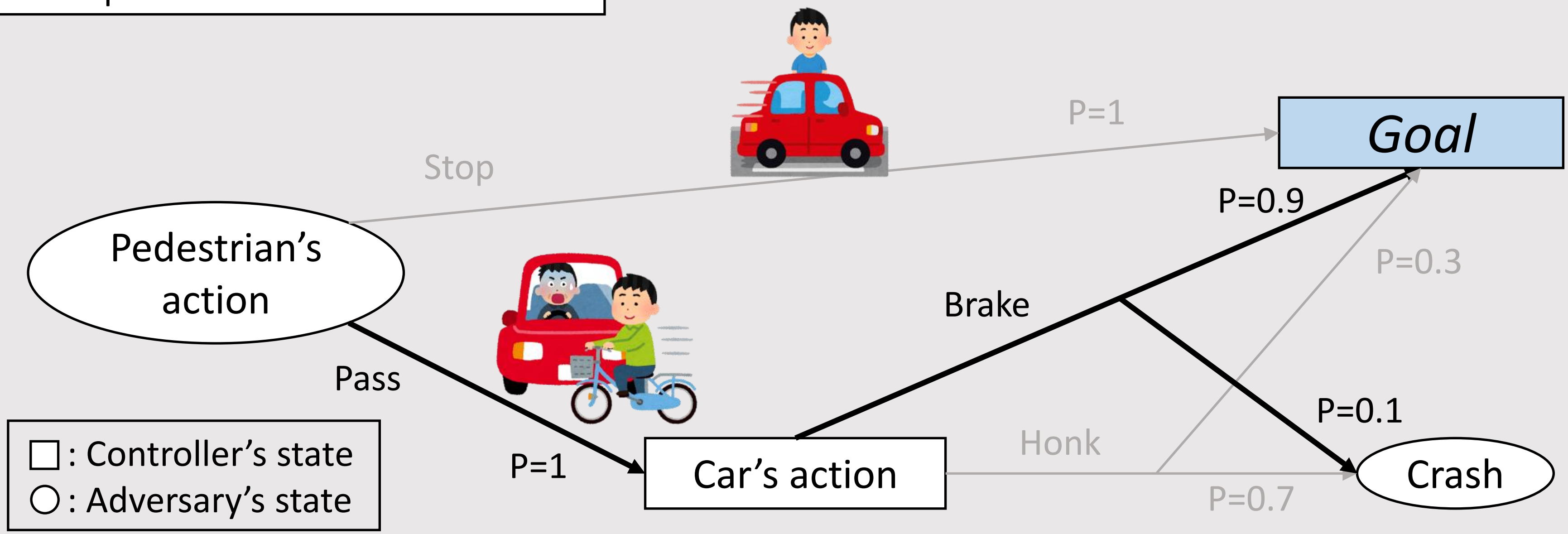

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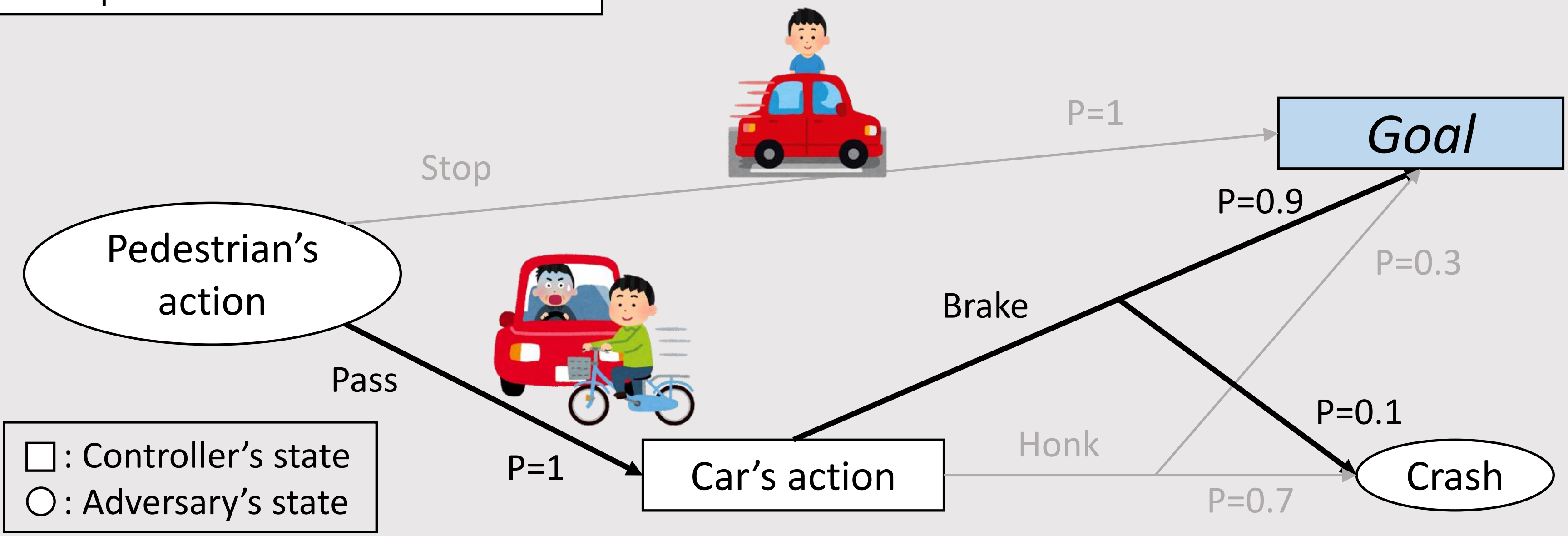

- A (pure positional) strategy of player X … σ: (X′s states) → (actions)
- Reachability prob. under strategies σ, τ of Controller/Adversary…

$$V_{\sigma,\tau}(s) = \Pr(\textit{Goal} \text{ is visited during the play, starting from } s, \text{under } \sigma \text{ and } \tau)$$

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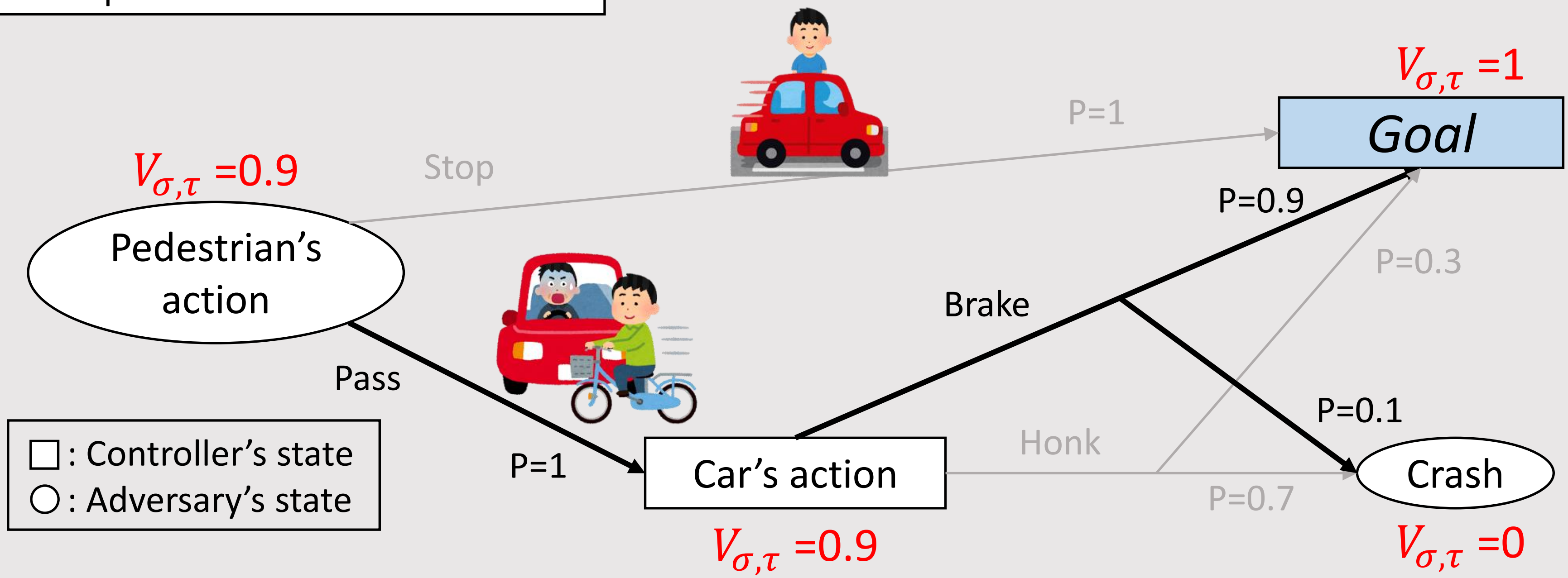

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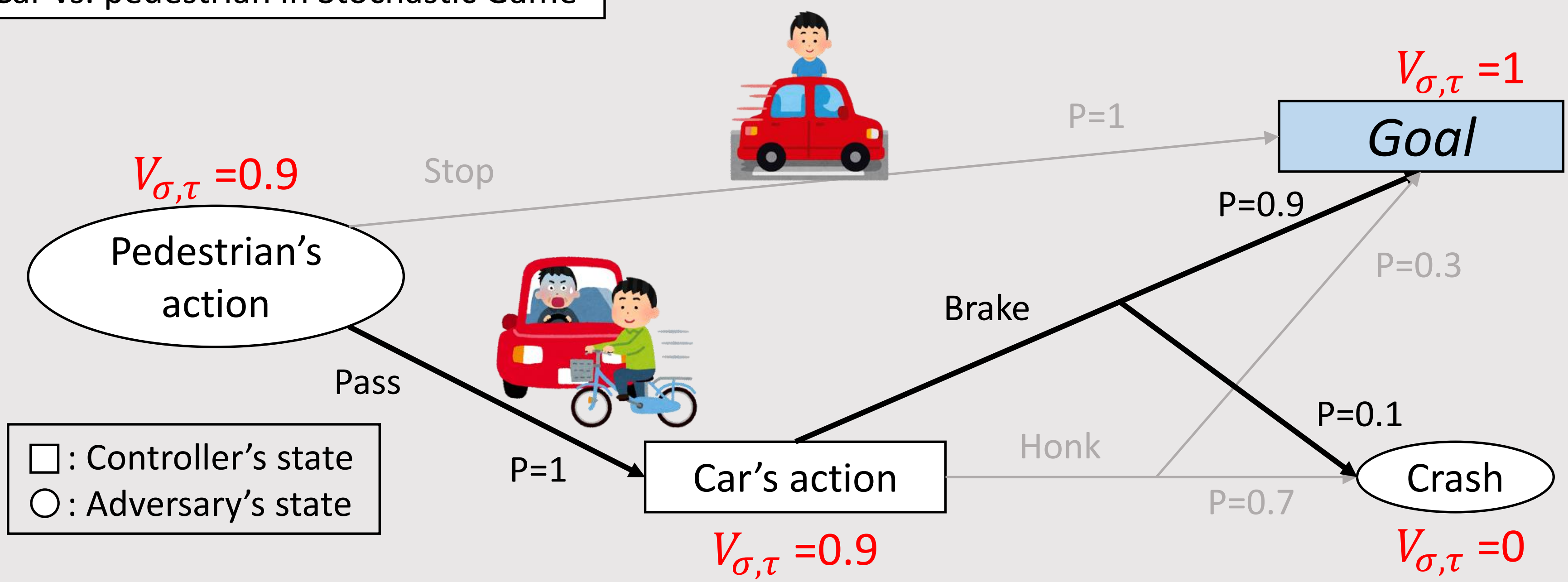

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Problem

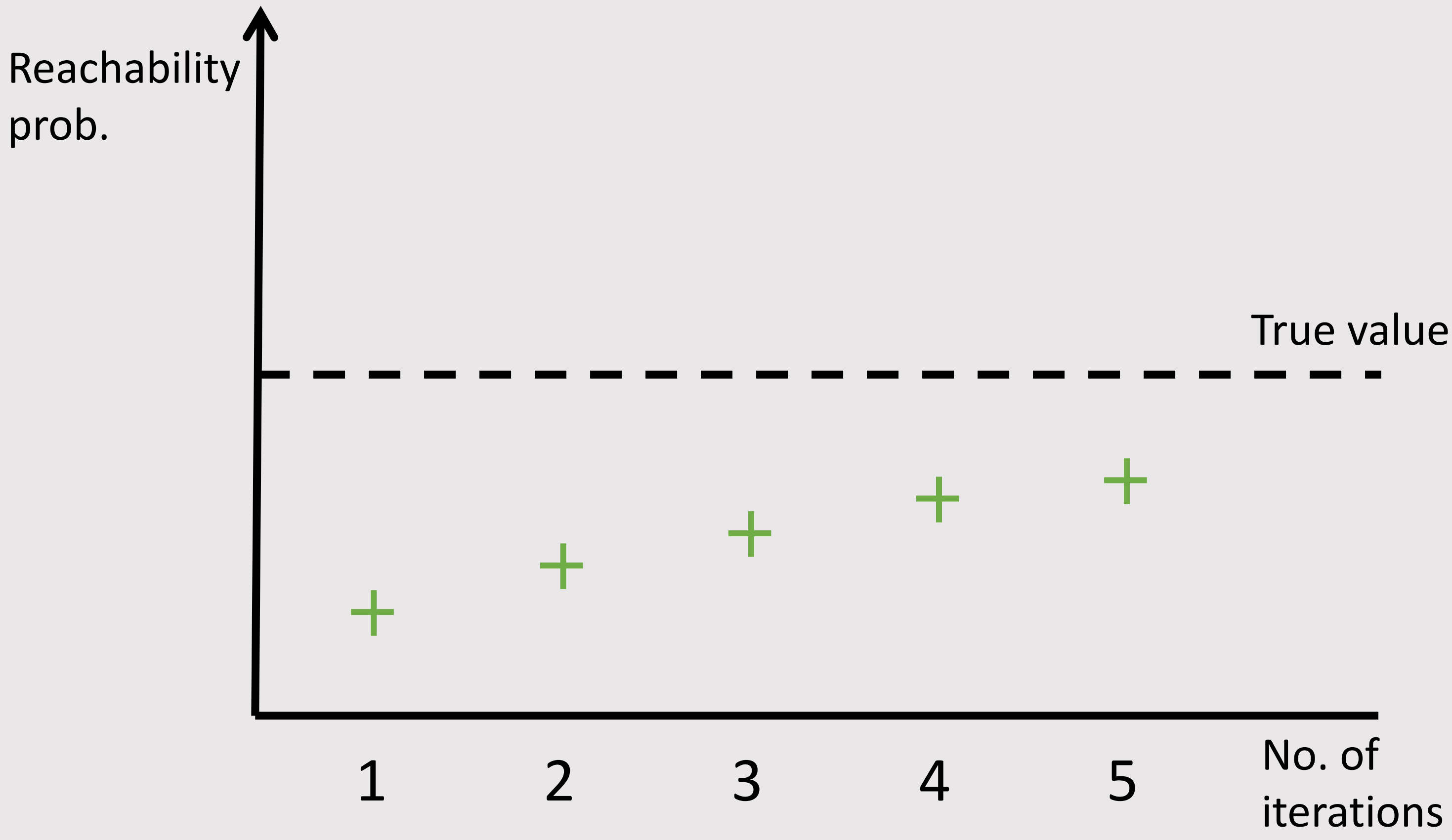
- Approximate the following $V\colon (\text{states}) \to [0,1]$

Controller/Adversary tries to maximize/minimize $V_{\sigma,\tau}$

$$V(s) = \max_{\sigma} \min_{\tau} V_{\sigma,\tau}(s)$$

Existing technique 1 : **Value Iteration (VI)**

- Generates an increasing sequence of lower bounds
- Converges to the true value
- No precision guarantee



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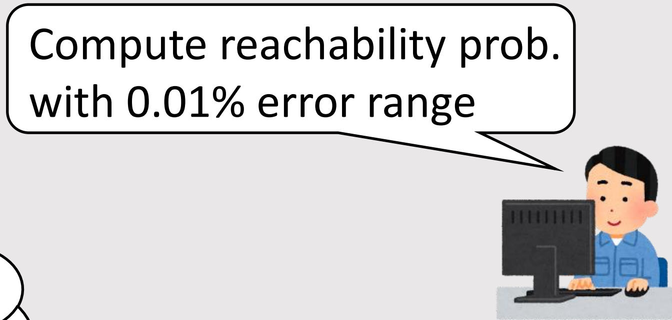

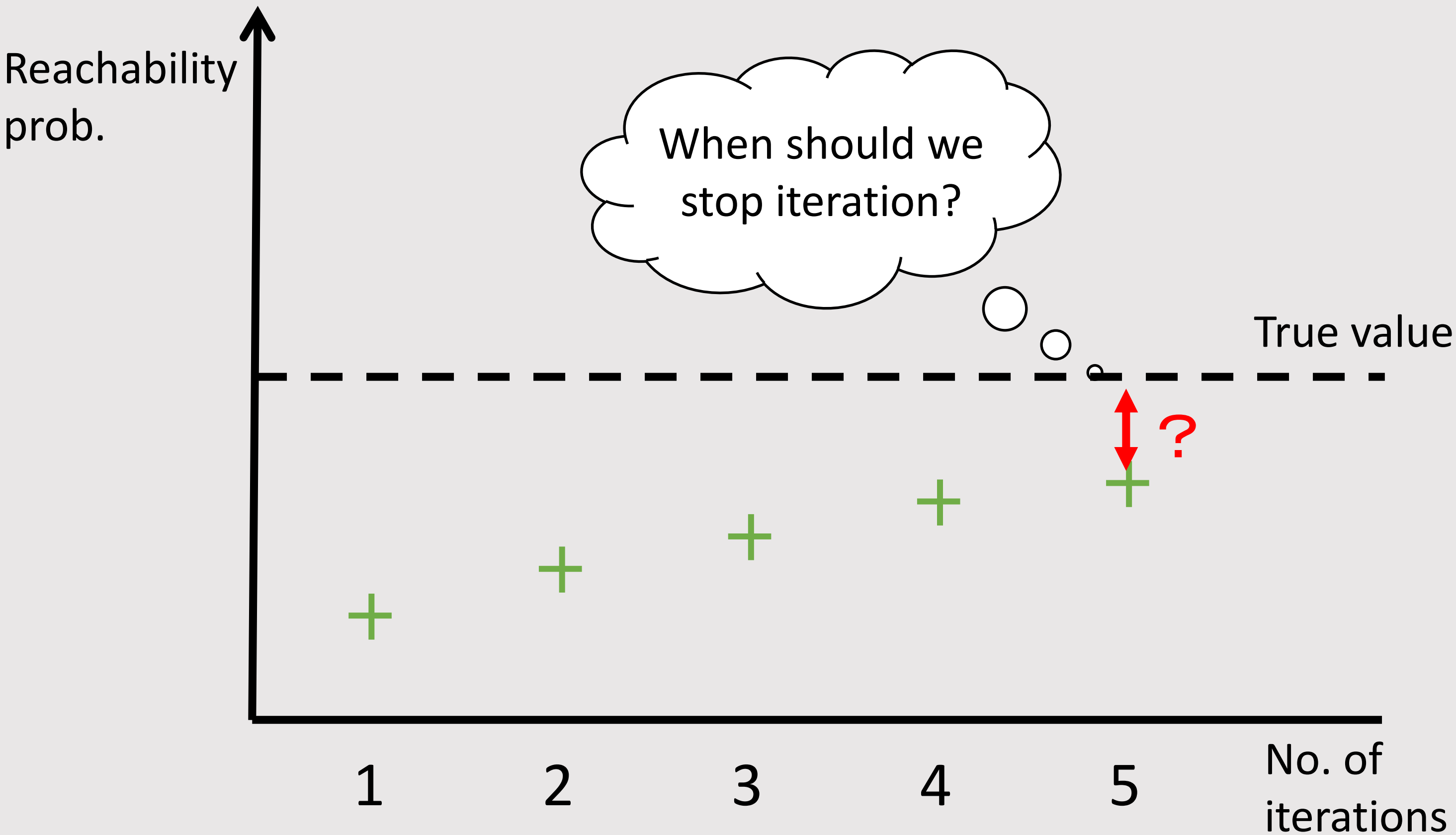

Existing technique 2 : **Bounded Value Iteration (BVI)**

[McMahan+,'05][Brazdil+,'14][Ujma, '15][Haddad+,'18][Kelmendi+,'18]

- Generates a decreasing sequence of upper bounds, too
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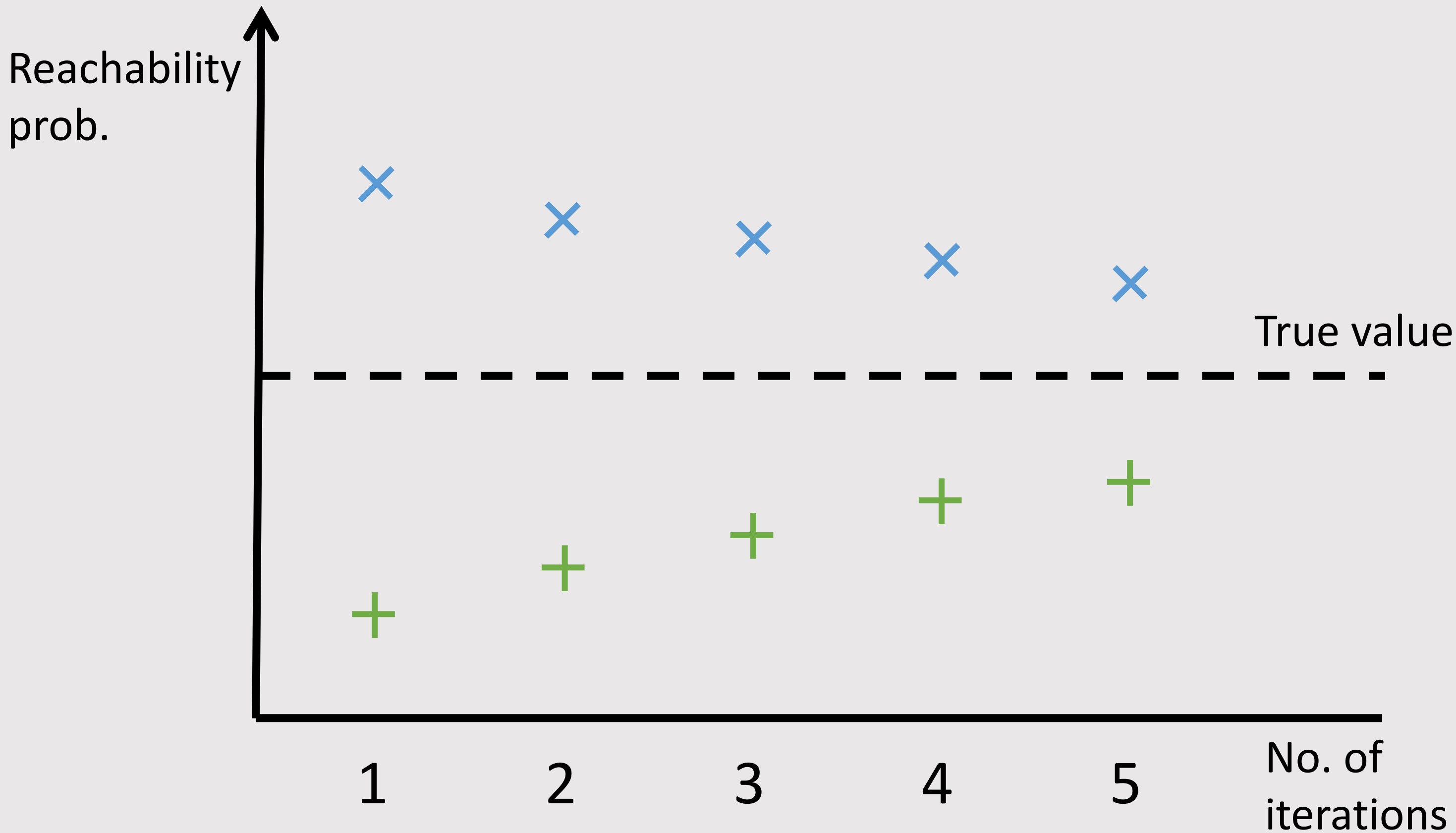

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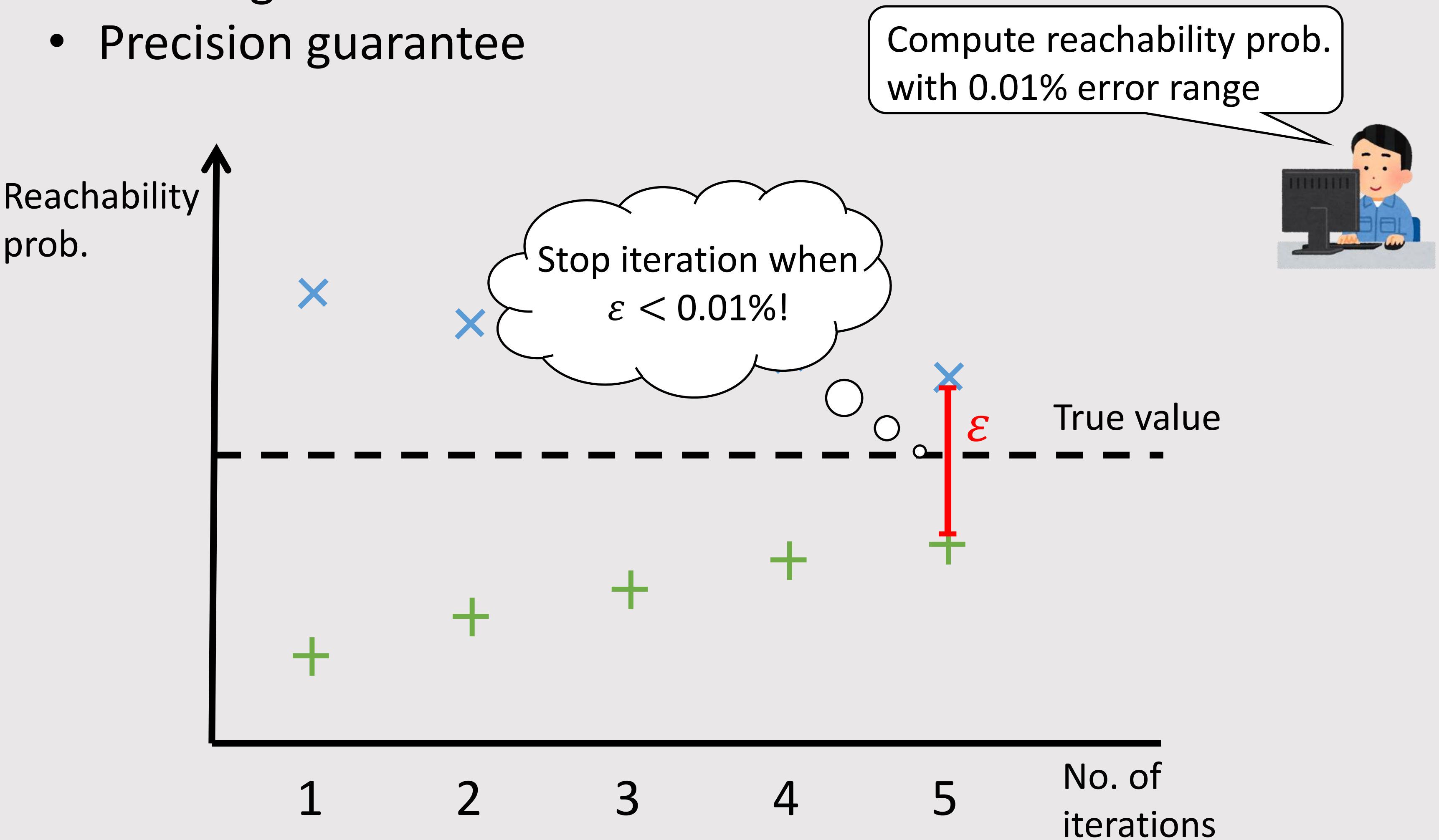


Technical challenge

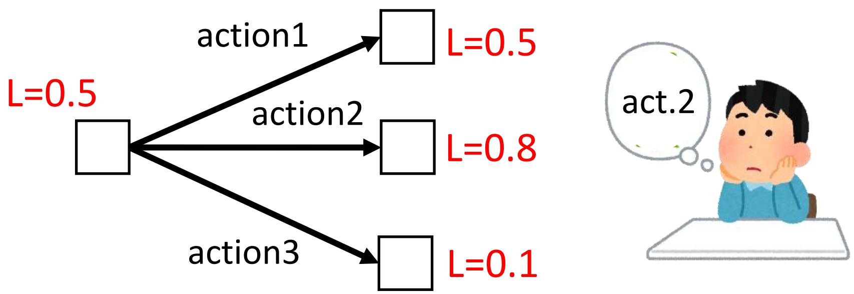
Q: Is BVI a technique that merely performs VI twice in parallel,
starting from some lower and upper bound?

A: No, it's more than that.
To assure convergence of upper bound, we need some trick.

How a lower bound L: (states) $\to [0,1]$ is updated via VI

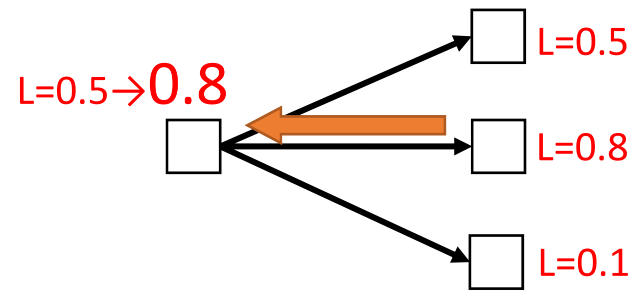
(at Controller's states)

Strategy estimation via local optimality



Choose an action that maximizes L after a transition

Bound update via estimated strategy



Back-propagate the value of L along the transition arrow

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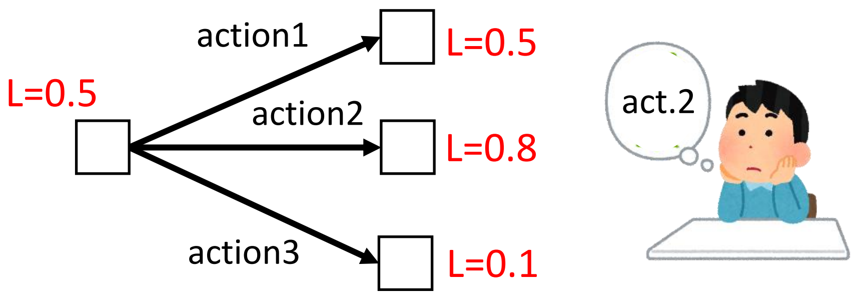

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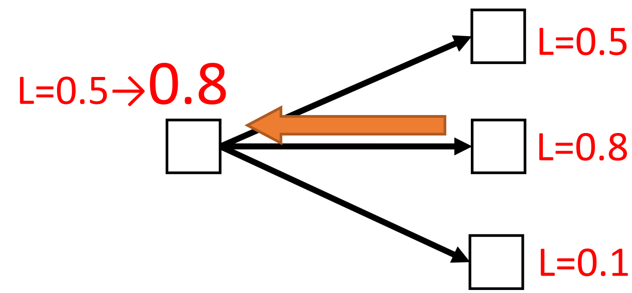

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Bellman operator $\mathbb{X}: L \mapsto \mathbb{X}L$

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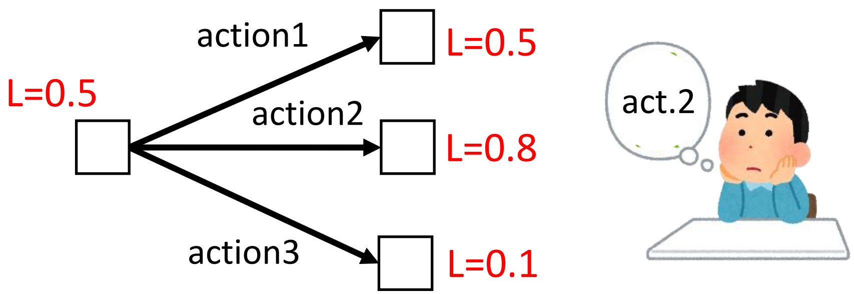

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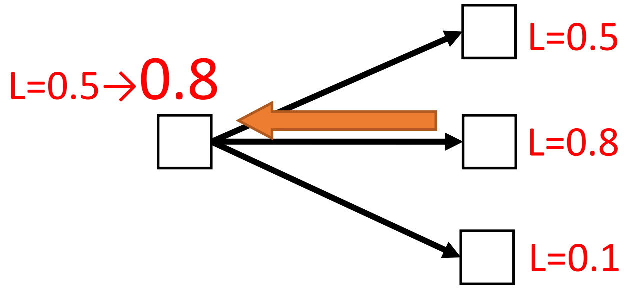

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VI performs $L_0 \mapsto \mathbb{X}L_0 \mapsto \mathbb{X}(\mathbb{X}L_0) \mapsto \mathbb{X}\big(\mathbb{X}(\mathbb{X}L_0)\big) \to \cdots$

How the non-convergence issue of an upper bound occurs

- Bellman operator is monotone over the set

$$\{f : (states) \to [0,1] \mid f(\mathrm{final}) = 1, f(\mathrm{sink}) = 0\}$$

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$f \le g \Leftrightarrow \forall s. f(s) \le g(s)$

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Convergence of lower bound

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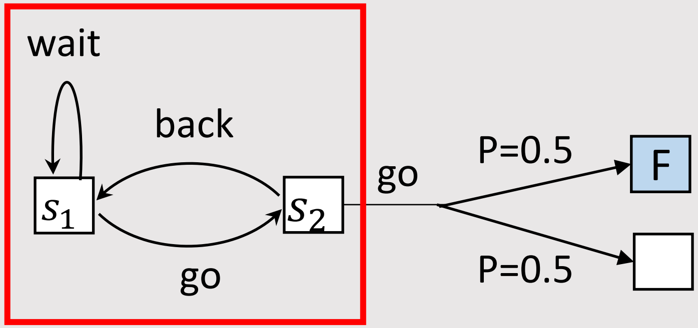
Existing technique to address the problem

- If the system is an MDP (i.e. there is no Adversary's state), GFP can be matched with LFP by merging *End Components* [Brazdil+,'14][Haddad+,'18]

Sub-MDP that constitutes a loop



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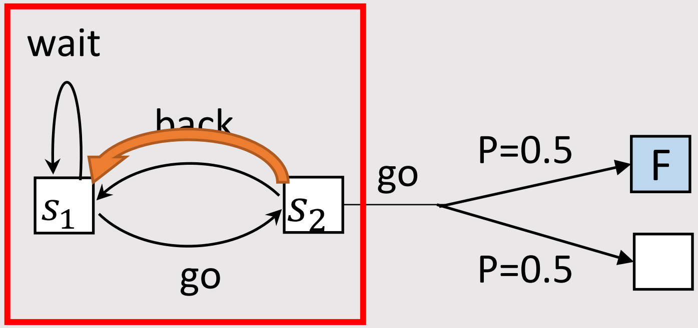

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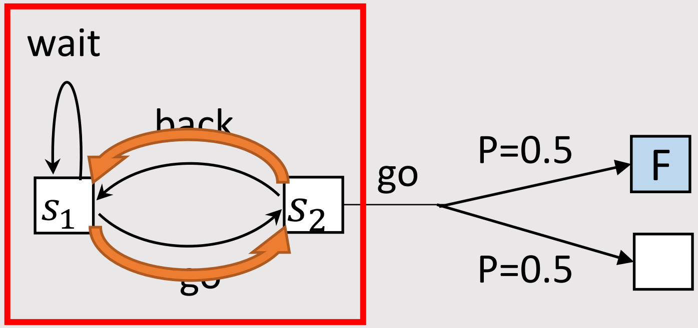

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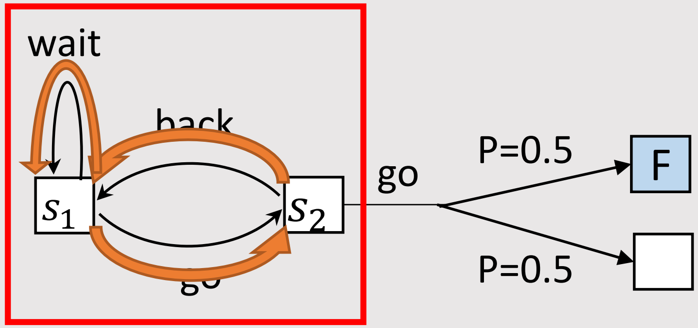

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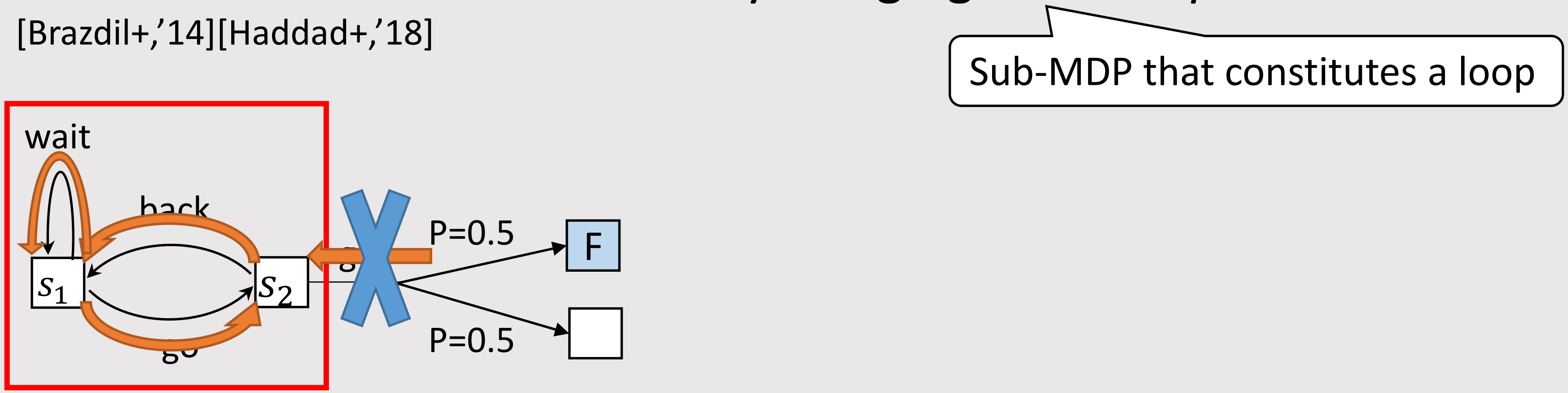

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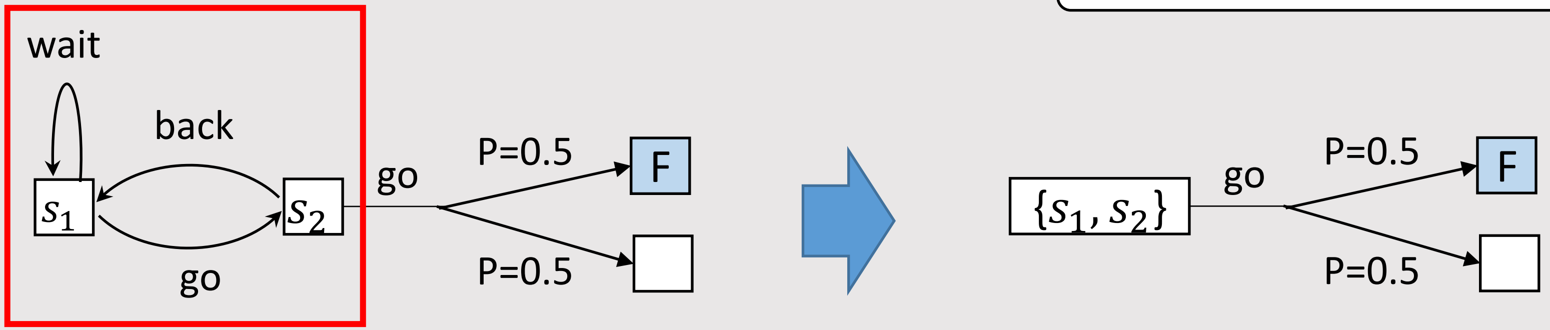

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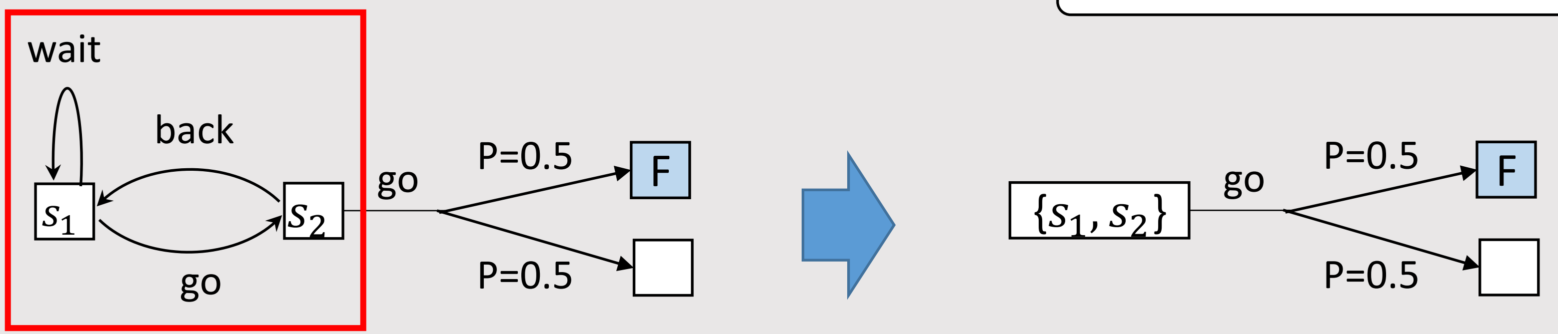

- For an arbitrary SG, we periodically *deflate* an upper bound while running the standard VI [Kelmendi+,'18]

Singular update of bound over (a sound approx. of) specific ECs

Overview of our algorithm

- Every existing technique involves EC computation
  (or restrict the model so that non-convergence problem does not occur)
- EC computation can be a bottleneck of execution time of BVI
  - Especially for SGs… EC computation is invoked many times

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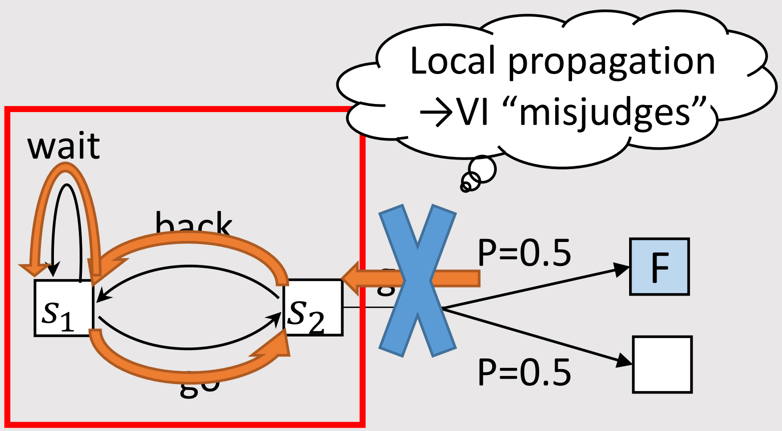
Our idea: ignore ECs, rather than compute

- Global propagation along the path to the final state



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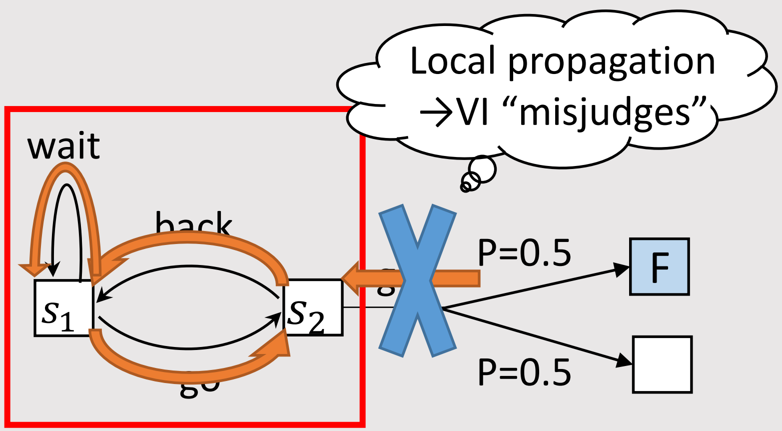
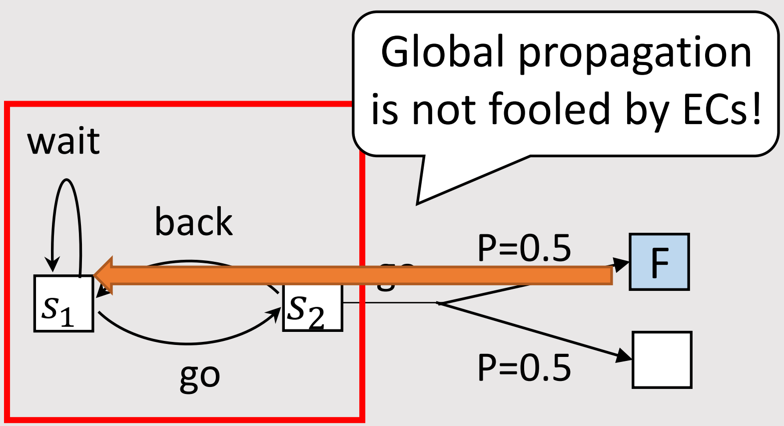
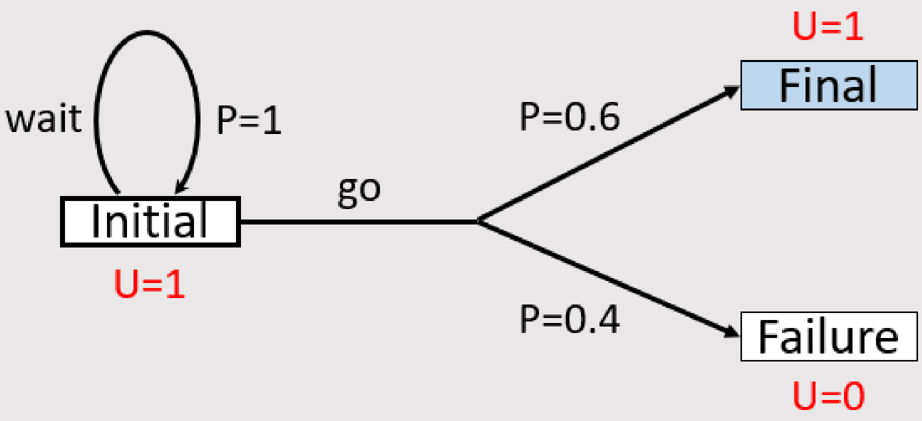




Step 1: Construct a weighted graph from MDP and upper bound U



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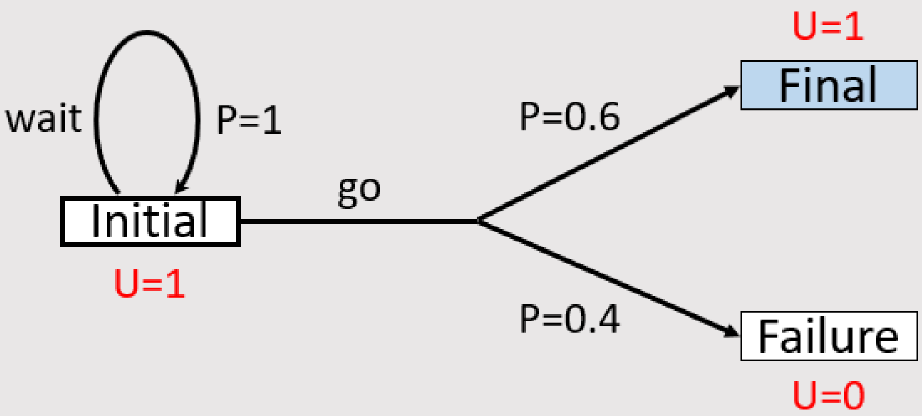

State	Action	Val. of U after a transition	Possible next state
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Initial	wait	1	Initial

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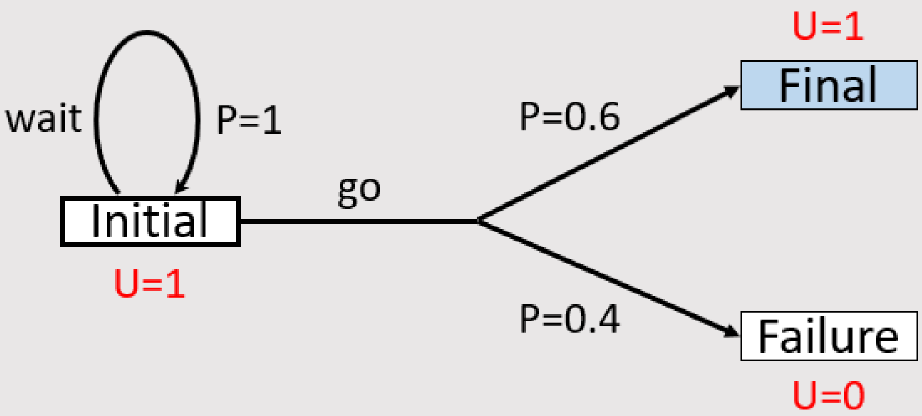

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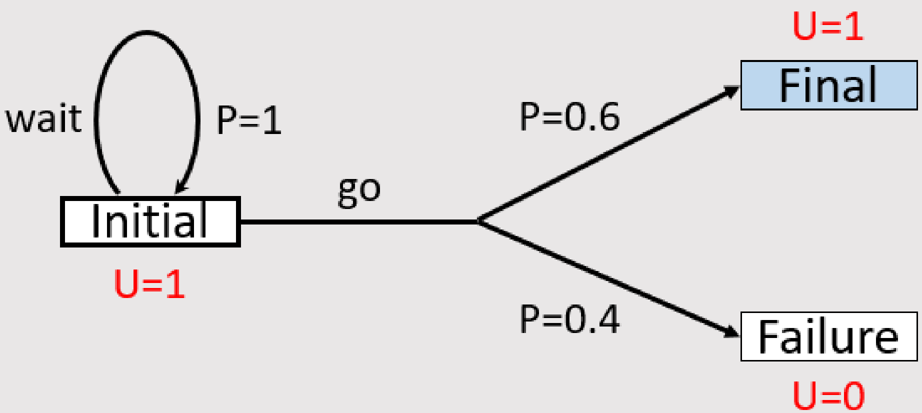

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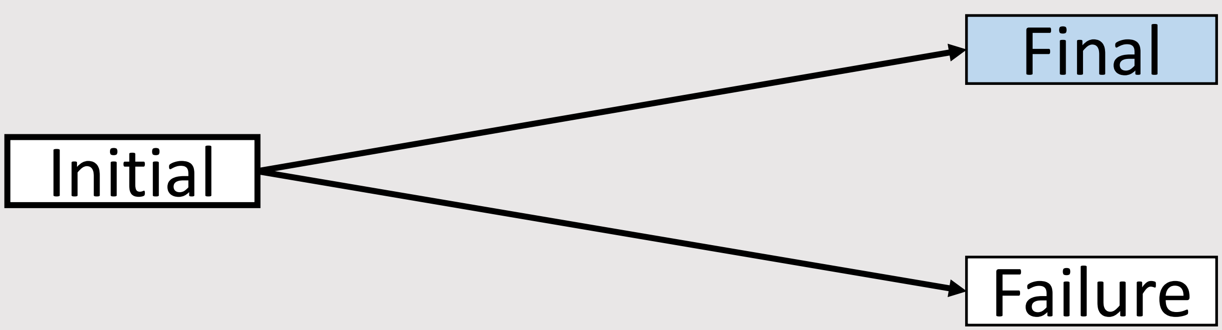

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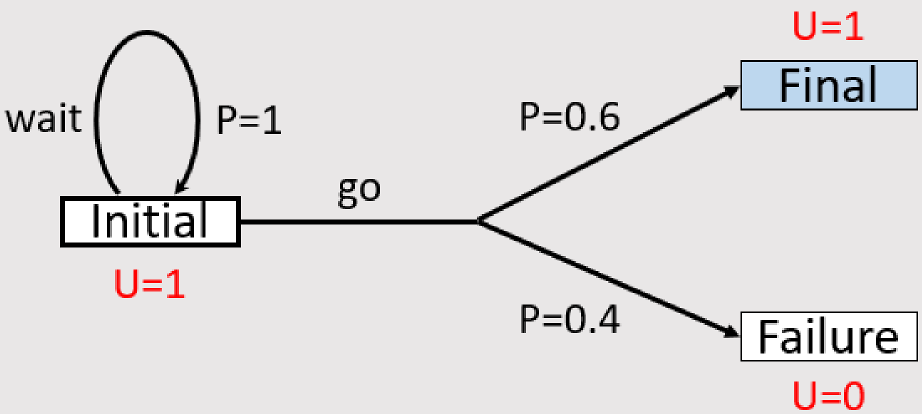

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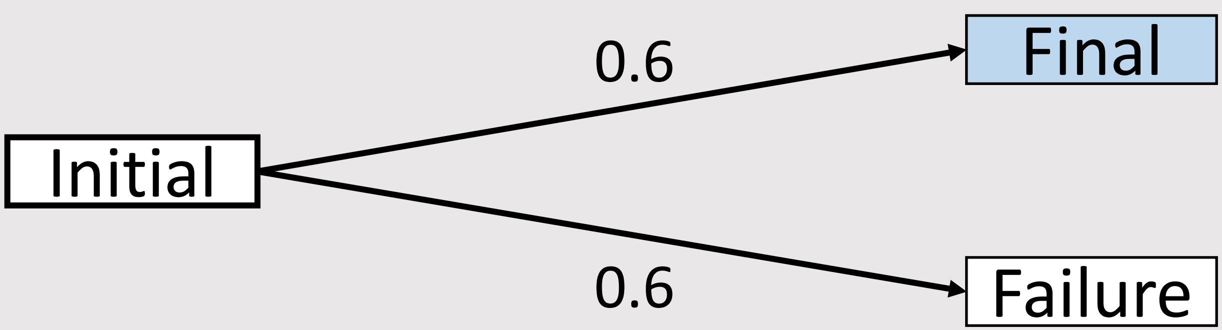

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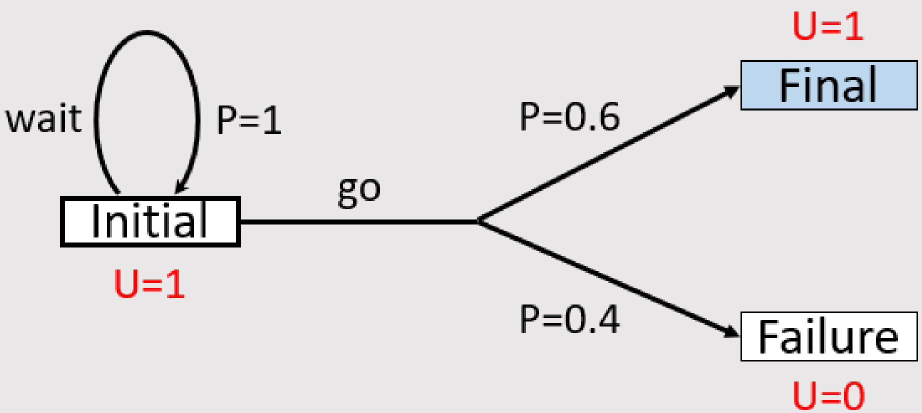

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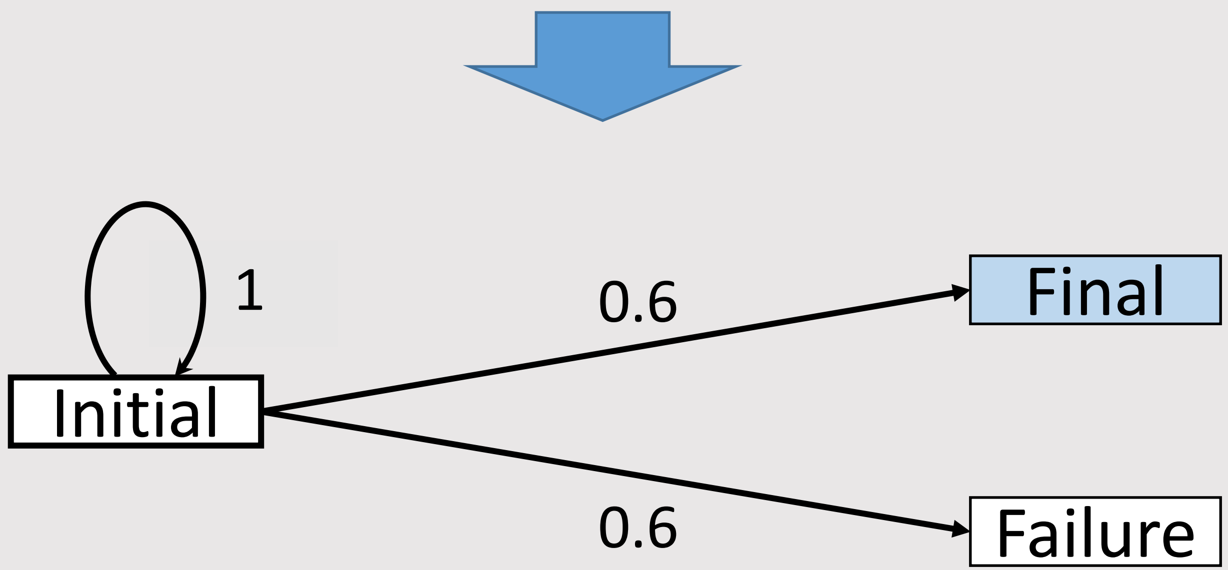
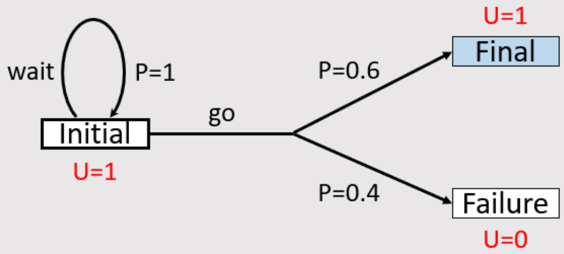
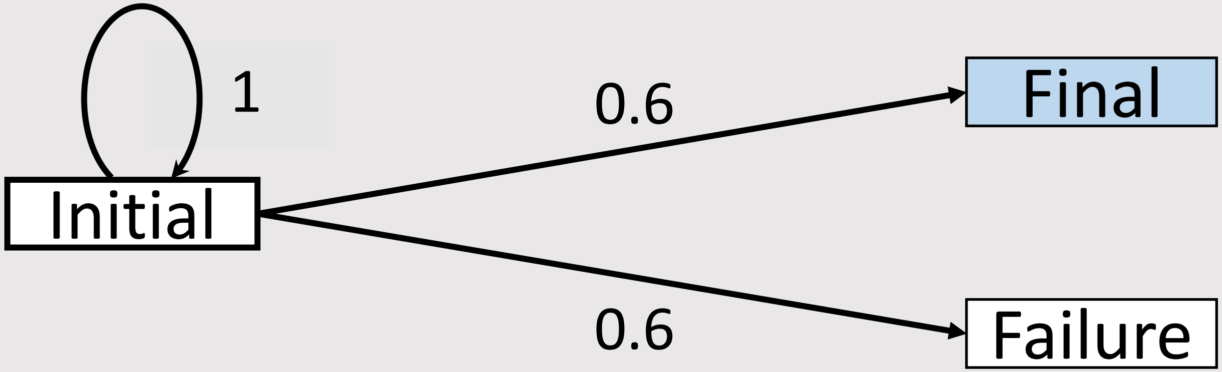
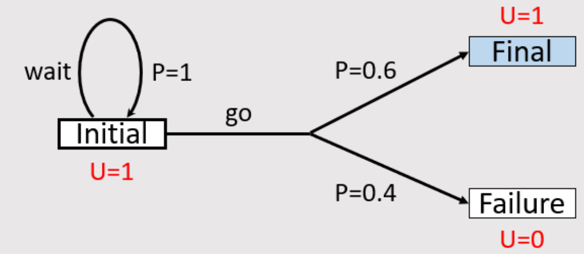


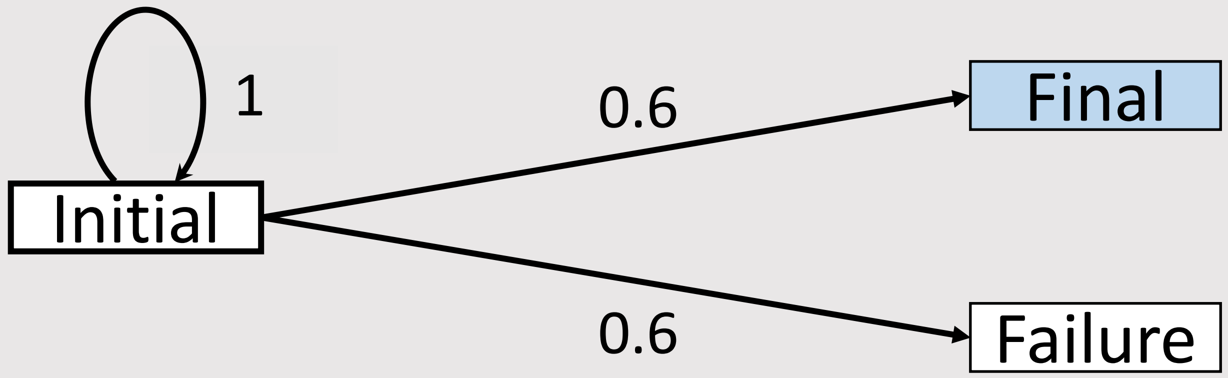
Step 2: update an upper bound U





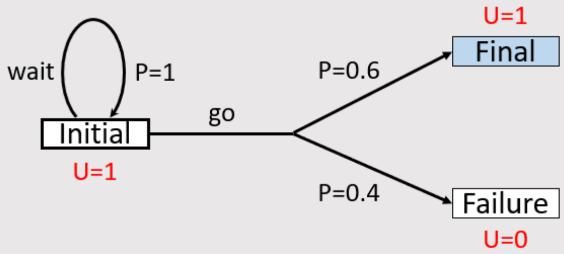
Step 2: update an upper bound U

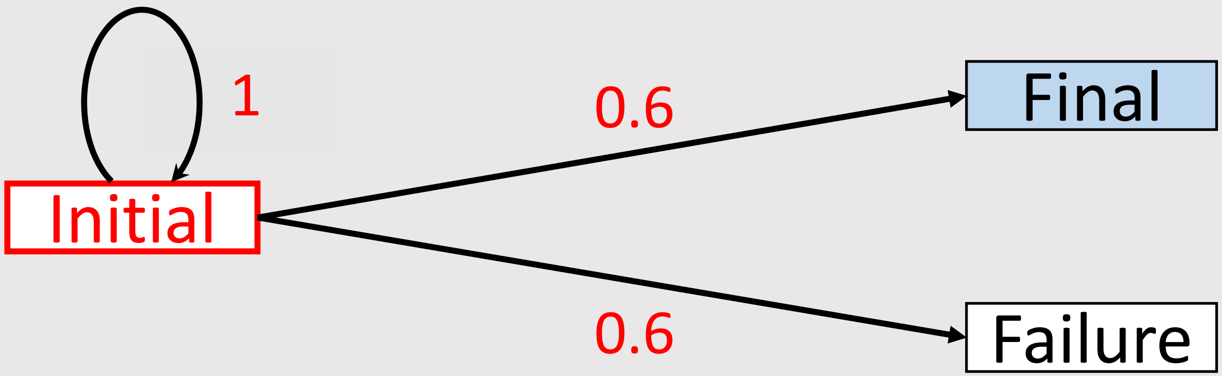




VI: “Compare outgoing edges, and propagate the largest weight”

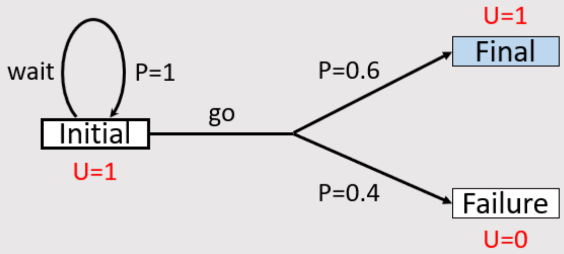
Step 2: update an upper bound U

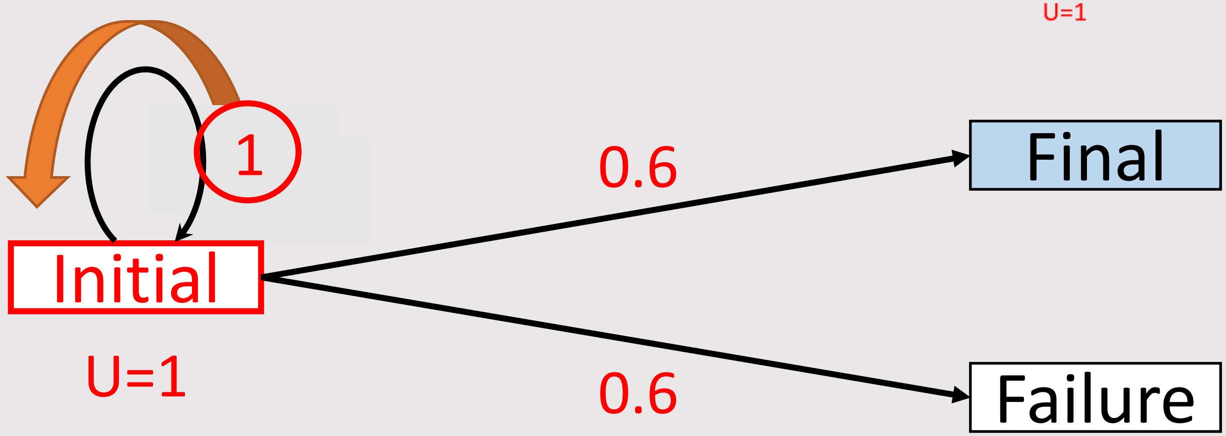




VI: “Compare outgoing edges, and propagate the largest weight”

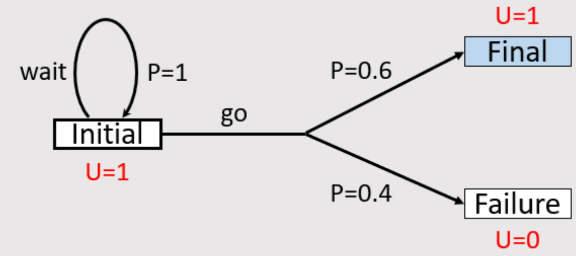
Step 2: update an upper bound U

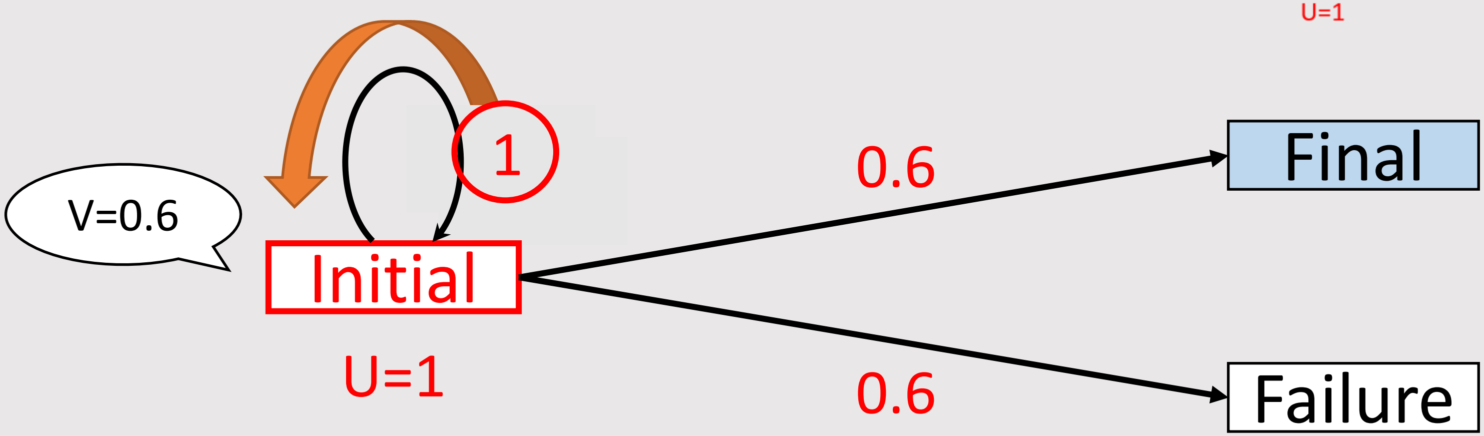




VI: “Compare outgoing edges, and propagate the largest weight”

Step 2: update an upper bound U





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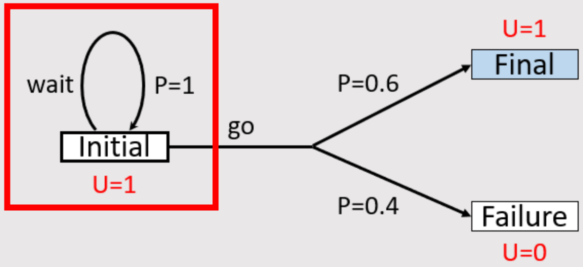
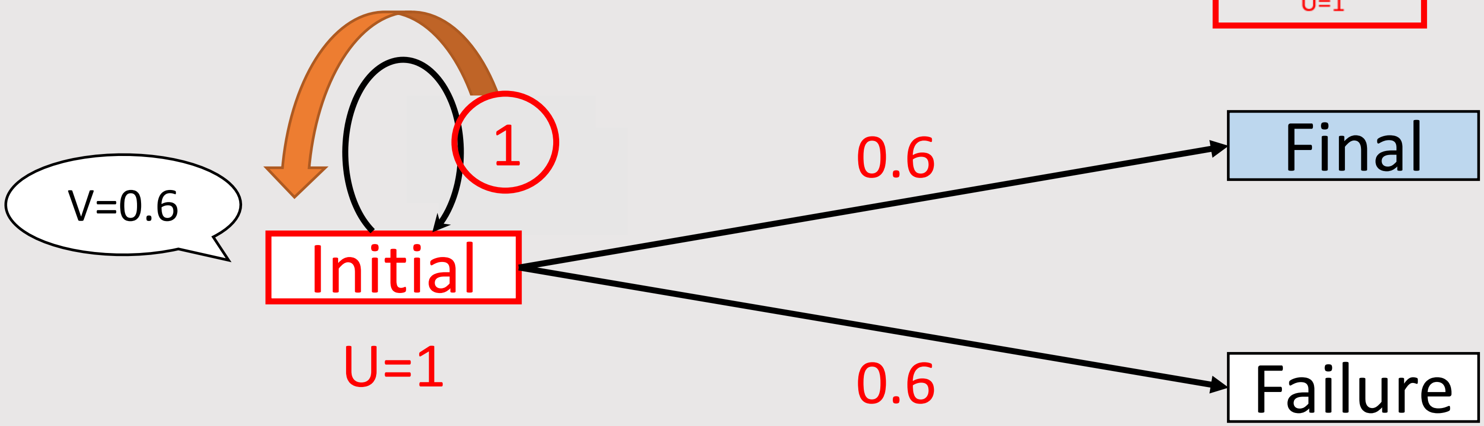
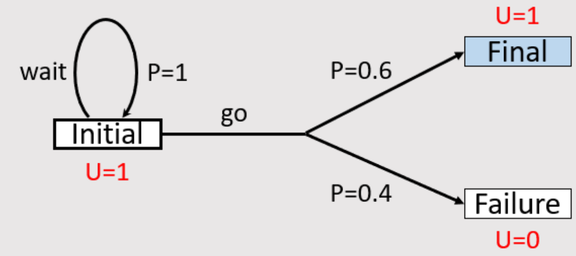
Step 2: update an upper bound U

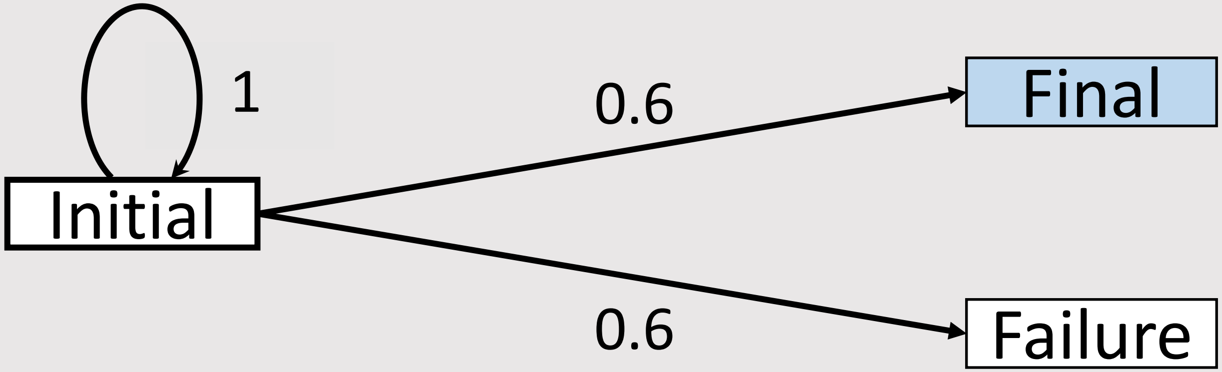




VI: “Compare outgoing edges, and propagate the largest weight”

Step 2: update an upper bound U

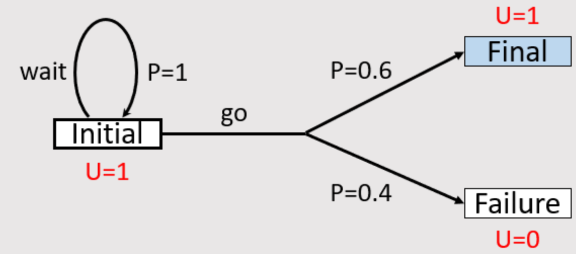


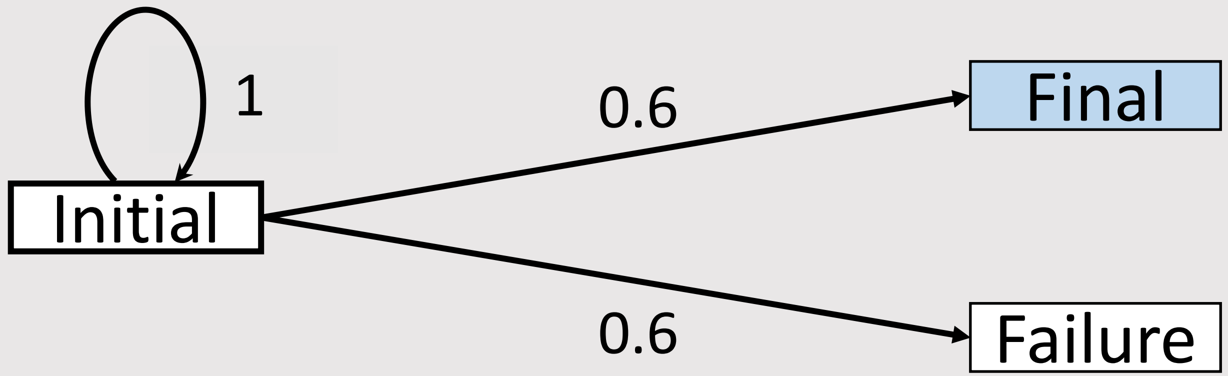


Our alg.: “Compare paths to Final, and propagate the largest width”

the minimum weight of constituting edges

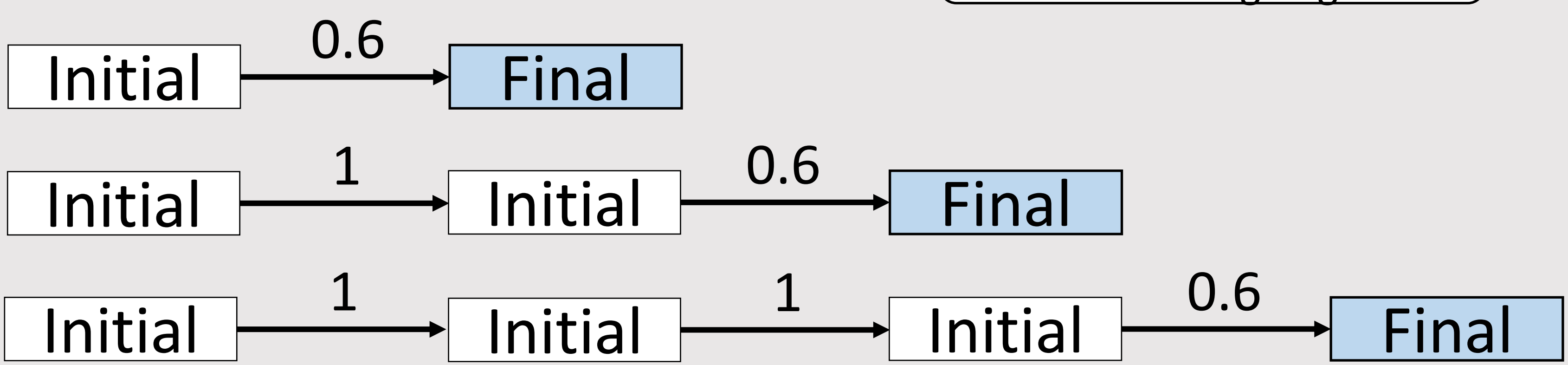
Step 2: update an upper bound U



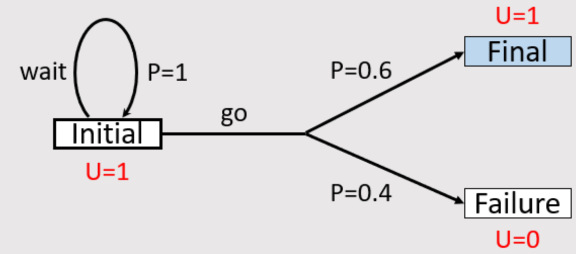


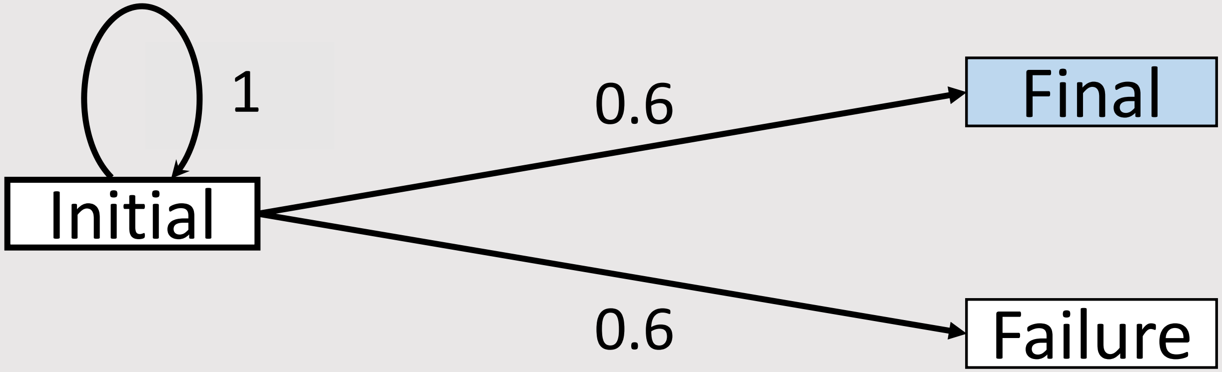
Our alg.: “Compare paths to Final, and propagate the largest width”

the minimum weight of constituting edges



Step 2: update an upper bound U





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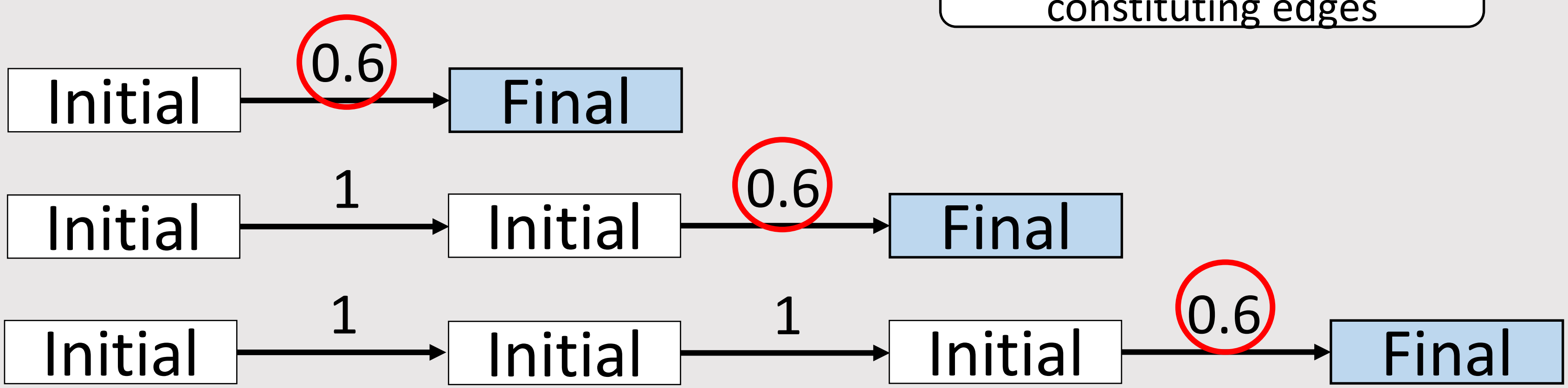

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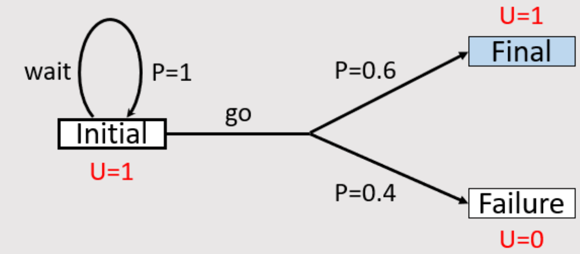
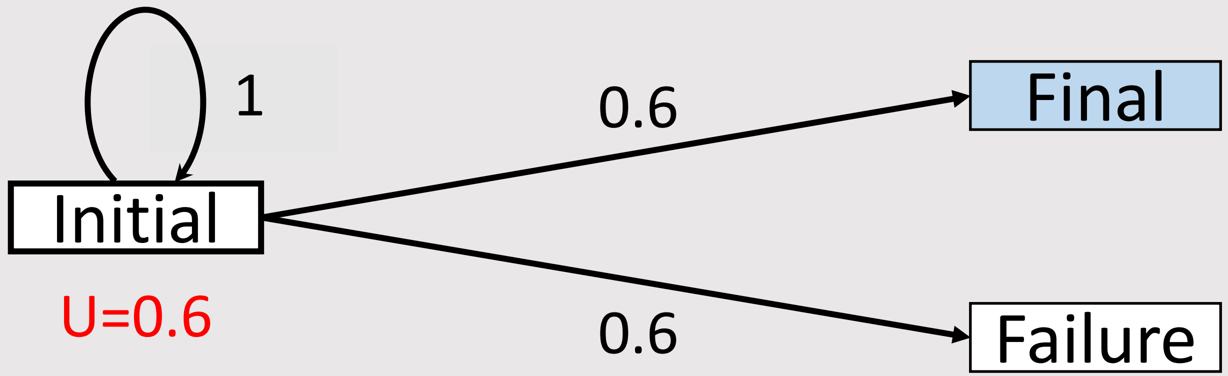
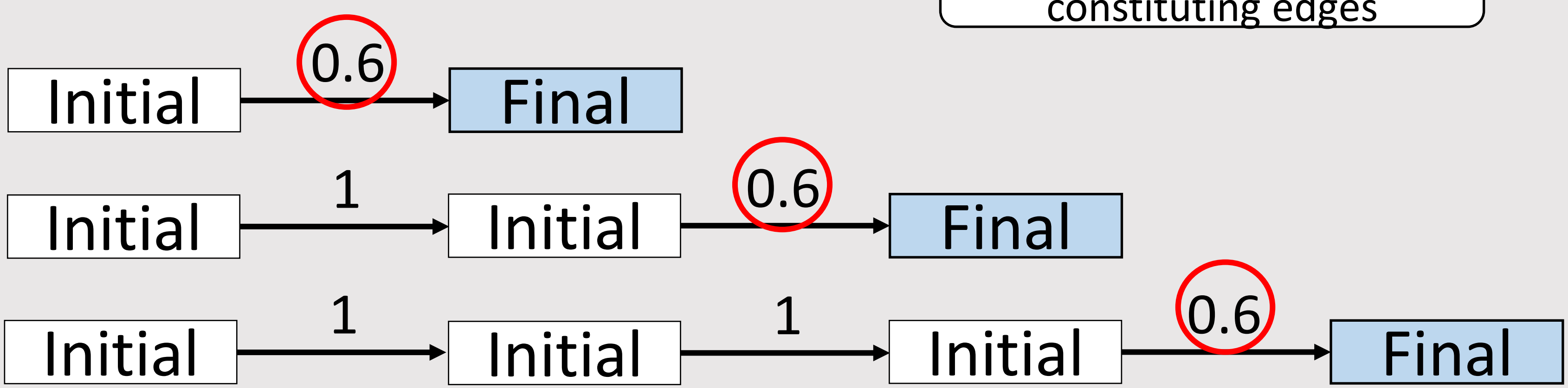


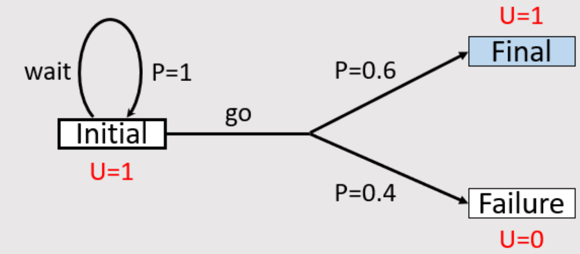


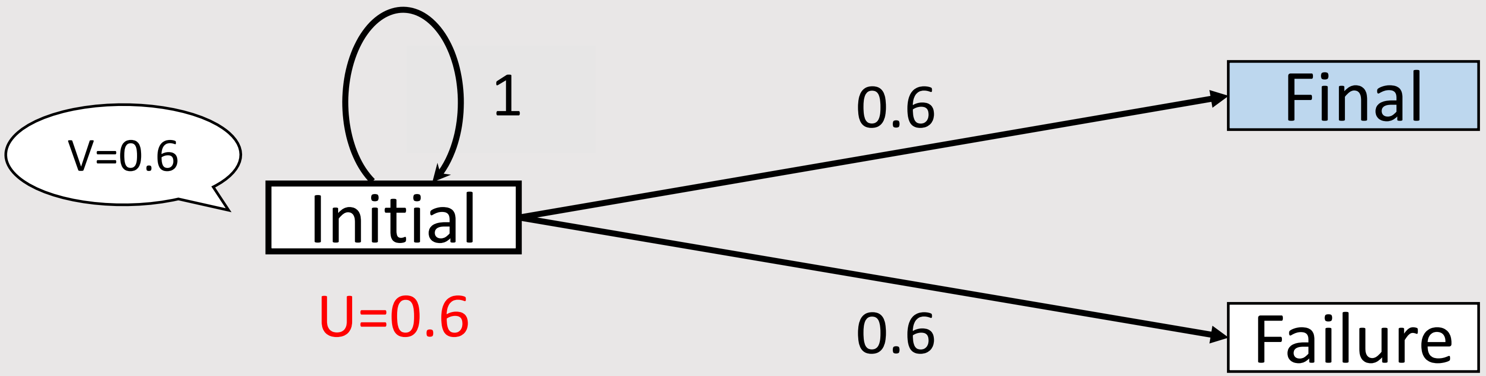
Our alg.: “Compare paths to Final, and propagate the largest width”

the minimum weight of constituting edges



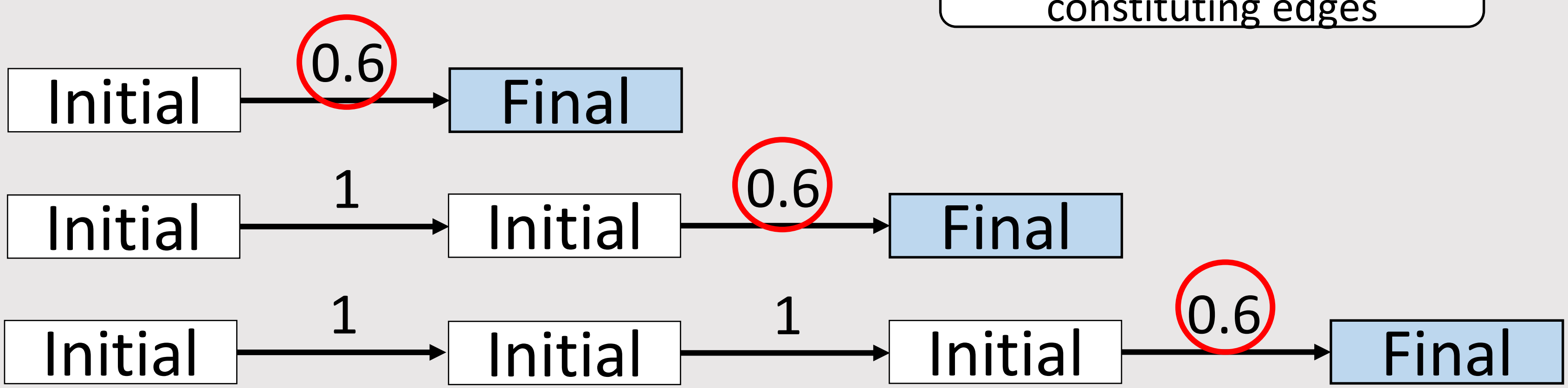
Step 2: update an upper bound U





Our alg.: “Compare paths to Final, and propagate the largest width”

the minimum weight of constituting edges



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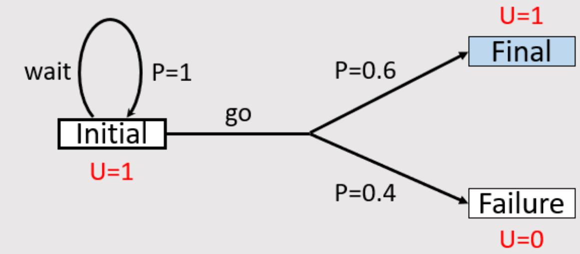
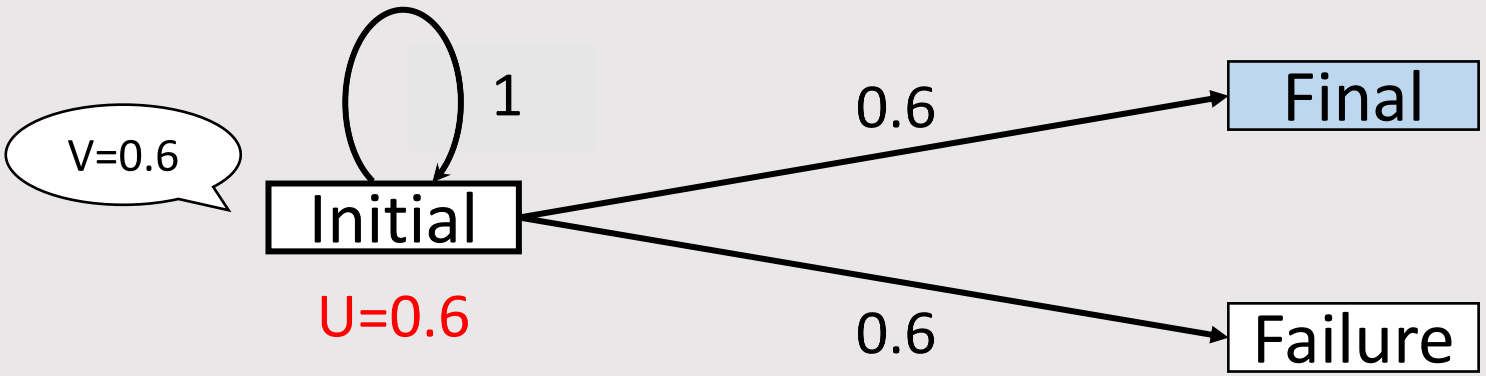
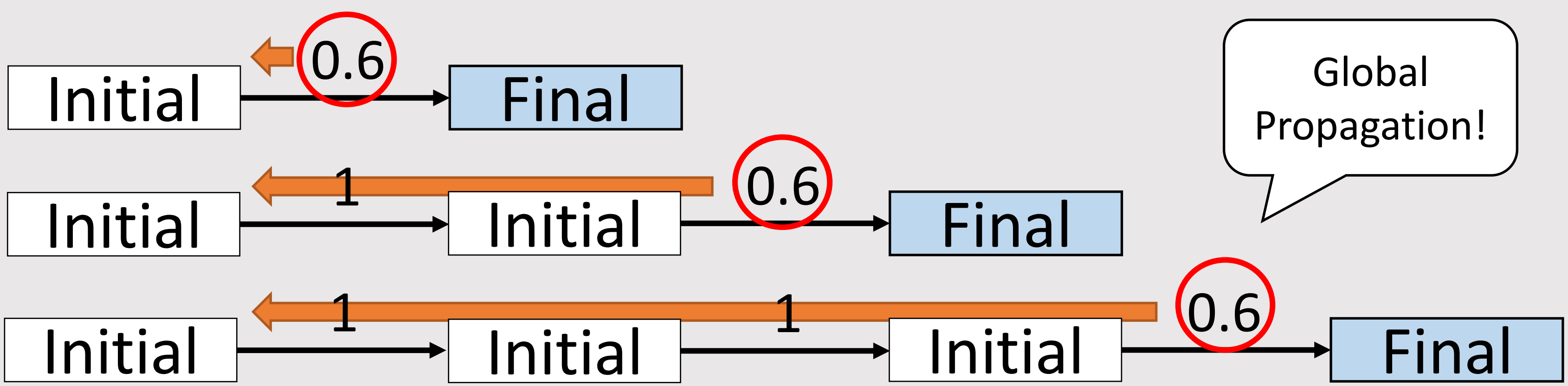
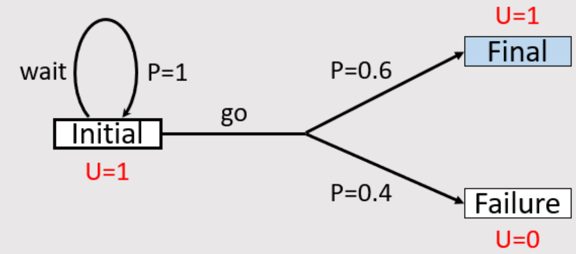


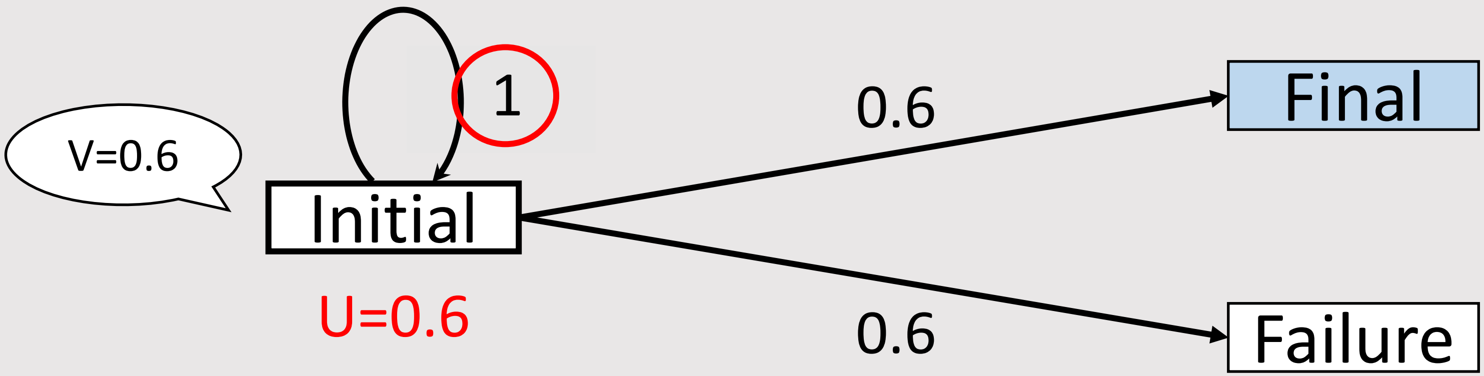


Our alg.: “Compare paths to Final, and propagate the largest width”

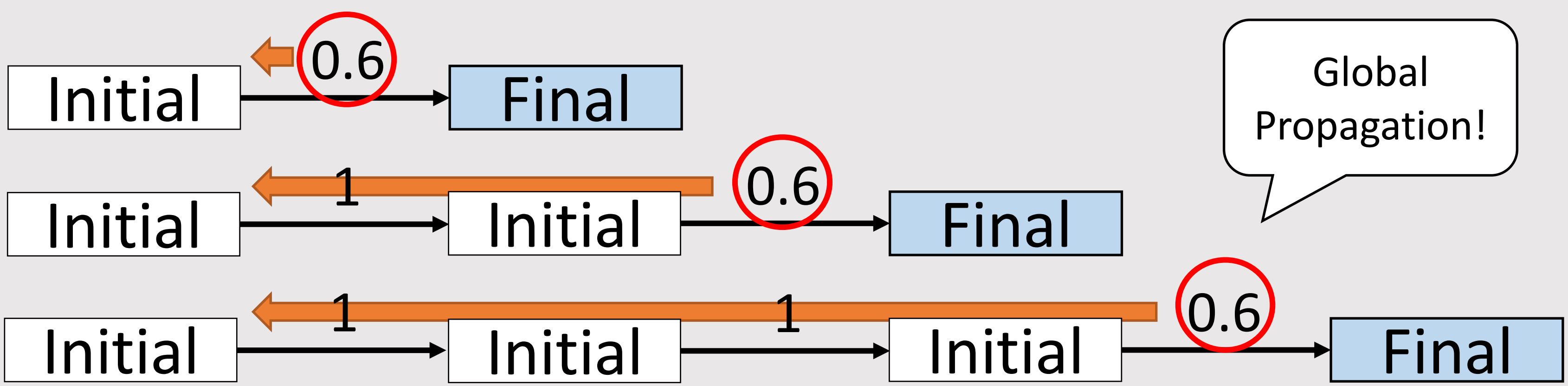


Step 2: update an upper bound U

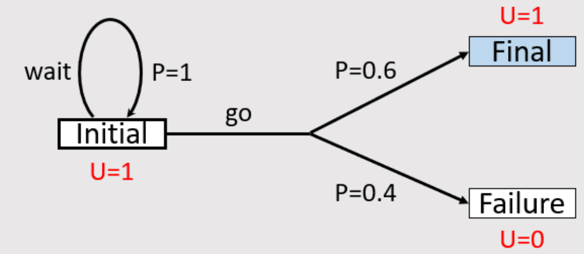


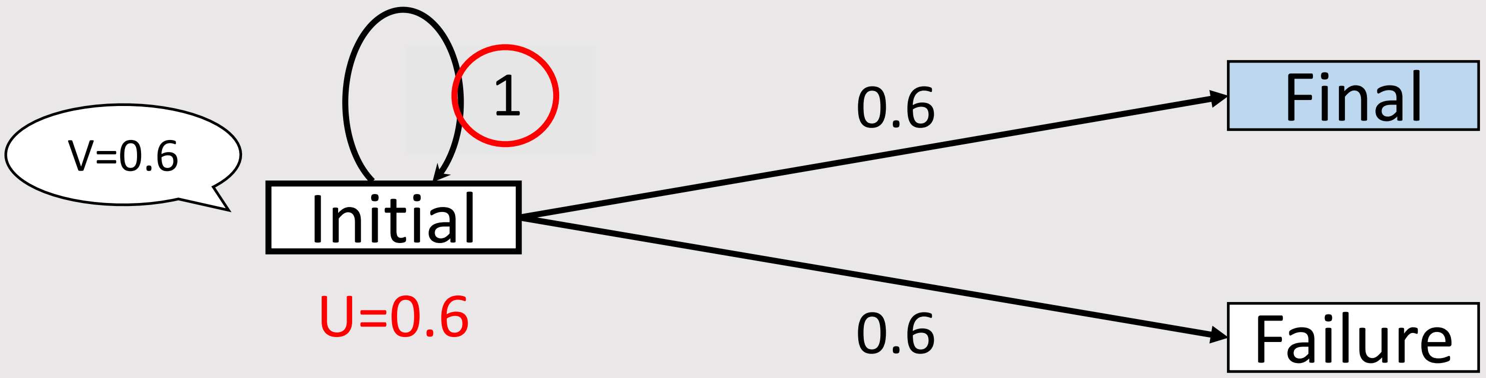


Our alg.: “Compare paths to Final, and propagate the largest width”

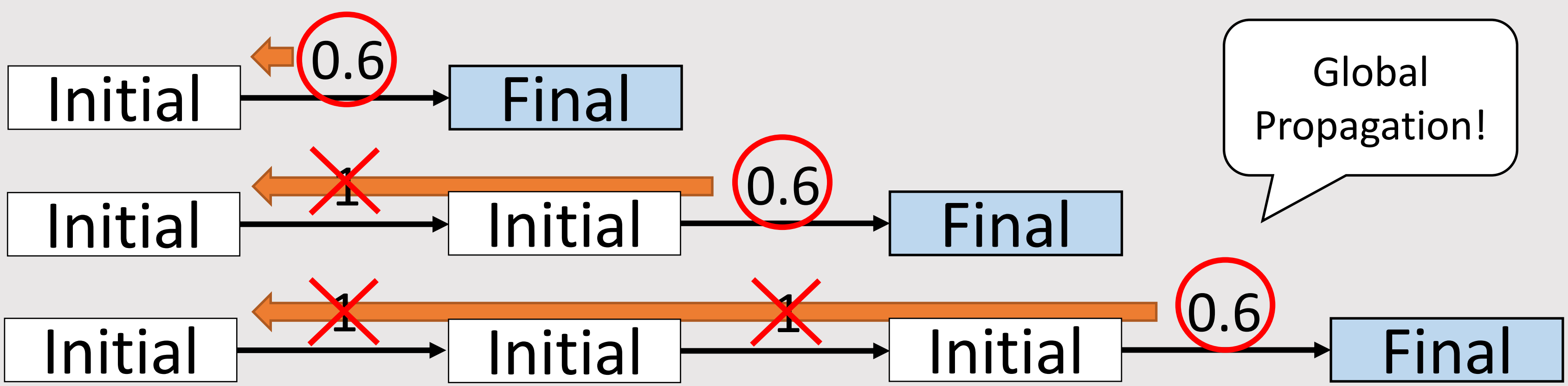


Step 2: update an upper bound U

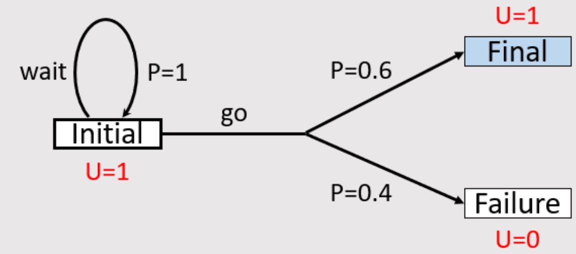


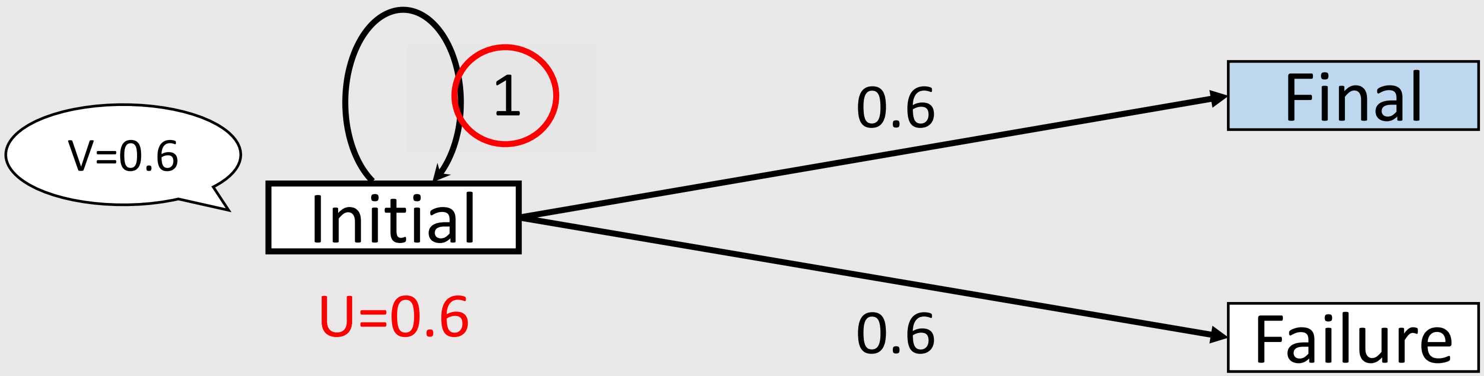


Our alg.: "Compare paths to Final, and propagate the largest width"

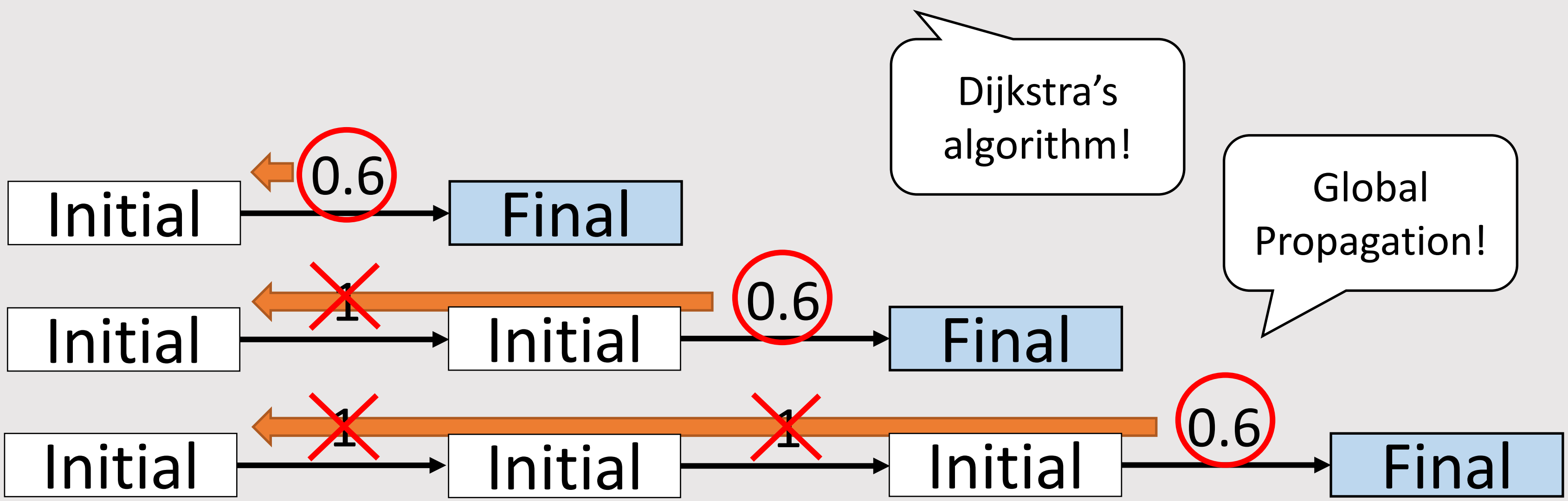


Step 2: update an upper bound U





Our alg.: "Compare paths to Final, and propagate the largest width"



Our algorithm for SGs

procedure $\mathrm{BVI\_WP}(\mathcal{G}, \varepsilon)$

$L_0 \leftarrow \bot,\ U_0 \leftarrow \top,\ i \leftarrow 0$

while $U_i(s_I) - L_i(s_I) > \varepsilon$ **do**

- $i{+}{+}$
- $L_i \leftarrow \mathbb{X}L_{i-1}$ `// value iteration for lower bounds`
- $\mathcal{M}_i \leftarrow \mathcal{M}_{\mathrm{PlRd}}(\mathcal{G}, L_i)$ `// player reduction`
- $\mathcal{W}_i \leftarrow \mathcal{W}_{\mathrm{LcPg}}(\mathcal{M}_i, U_{i-1})$ `// local propagation`
- $U_i \leftarrow \min\{U_{i-1}, \mathrm{WPW}(\mathcal{W}_i)\}$ `// widest path computation`

return $U_i(s_I)$

Our algorithm for SGs

procedure $\underline{\mathrm{BVI\_WP}(\mathcal{G}, \varepsilon)}$ $\mathcal{G}$: SG, ε: precision requirement

$L_0 \leftarrow \bot,\ U_0 \leftarrow \top,\ i \leftarrow 0$

while $U_i(s_I) - L_i(s_I) > \varepsilon$ **do**

$i{+}{+}$

$L_i \leftarrow \mathbb{X} L_{i-1}$ // value iteration for lower bounds

$\mathcal{M}_i \leftarrow \mathcal{M}_{\mathrm{PlRd}}(\mathcal{G}, L_i)$ // player reduction

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$U_i \leftarrow \min\{U_{i-1}, \mathrm{WPW}(\mathcal{W}_i)\}$ // widest path computation

return $U_i(s_I)$

Our algorithm for SGs

```
procedure BVI_WP(𝒢, ε)
    L_0 ← ⊥, U_0 ← ⊤, i ← 0
    while U_i(s_I) − L_i(s_I) > ε do
        i++
        L_i ← 𝕏L_{i−1}                          // value iteration for lower bounds
        ℳ_i ← ℳ_PlRd(𝒢, L_i)                    // player reduction
        𝒲_i ← 𝒲_LcPg(ℳ_i, U_{i−1})              // local propagation
        U_i ← min{U_{i−1}, WPW(𝒲_i)}            // widest path computation
    return U_i(s_I)
```

$\mathcal{G}$: SG, ε: precision requirement

Our algorithm for SGs

procedure $\mathrm{BVI\_WP}(\mathcal{G}, \varepsilon)$ — $\mathcal{G}$: SG, ε: precision requirement

$L_0 \leftarrow \bot,\ U_0 \leftarrow \top,\ i \leftarrow 0$

while $U_i(s_I) - L_i(s_I) > \varepsilon$ **do**

$i{+}{+}$

$L_i \leftarrow \mathbb{X} L_{i-1}$ // value iteration for lower bounds

$\mathcal{M}_i \leftarrow \mathcal{M}_{\mathrm{PlRd}}(\mathcal{G}, L_i)$ // player reduction

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$U_i \leftarrow \min\{U_{i-1}, \mathrm{WPW}(\mathcal{W}_i)\}$ // widest path computation

return $U_i(s_I)$

Our algorithm for SGs

procedure BVI_WP$(\mathcal{G}, \varepsilon)$ — $\mathcal{G}$: SG, ε: precision requirement

  $L_0 \leftarrow \bot,\ U_0 \leftarrow \top,\ i \leftarrow 0$

  while $U_i(s_I) - L_i(s_I) > \varepsilon$ **do**

    $i{+}{+}$

    $L_i \leftarrow \mathbb{X} L_{i-1}$ `// value iteration for lower bounds`

    MDP $\mathcal{M}_i \leftarrow \mathcal{M}_{\mathrm{PlRd}}(\mathcal{G}, L_i)$ `// player reduction`

    $\mathcal{W}_i \leftarrow \mathcal{W}_{\mathrm{LcPg}}(\mathcal{M}_i, U_{i-1})$ `// local propagation`

    $U_i \leftarrow \min\{U_{i-1}, \mathrm{WPW}(\mathcal{W}_i)\}$ `// widest path computation`

  return $U_i(s_I)$

Our algorithm for SGs

procedure BVI_WP($\mathcal{G}, \varepsilon$)　　$\mathcal{G}$: SG, ε: precision requirement

　$L_0 \leftarrow \bot,\ U_0 \leftarrow \top,\ i \leftarrow 0$

　while $U_i(s_I) - L_i(s_I) > \varepsilon$ **do**

　　i++

　　$L_i \leftarrow \mathbb{X} L_{i-1}$　　`// value iteration for lower bounds`

　　$\mathcal{M}_i \leftarrow \mathcal{M}_{\mathrm{PlRd}}(\mathcal{G}, L_i)$　　`// player reduction`

　　$\mathcal{W}_i \leftarrow \mathcal{W}_{\mathrm{LcPg}}(\mathcal{M}_i, U_{i-1})$　　`// local propagation`

　　$U_i \leftarrow \min\{U_{i-1}, \mathrm{WPW}(\mathcal{W}_i)\}$　　`// widest path computation`

　return $U_i(s_I)$

MDP

For every sufficiently large i,
reachability prob. of $\mathcal{M}_i$ and $\mathcal{G}$ are the same

Our algorithm for SGs

```
procedure BVI_WP(𝒢, ε)
    L_0 ← ⊥, U_0 ← ⊤, i ← 0
    while U_i(s_I) − L_i(s_I) > ε do
        i++
        L_i ← 𝕏L_{i−1}                          // value iteration for lower bounds
        ℳ_i ← ℳ_PlRd(𝒢, L_i)                    // player reduction
        𝒲_i ← 𝒲_LcPg(ℳ_i, U_{i−1})              // local propagation
        U_i ← min{U_{i−1}, WPW(𝒲_i)}            // widest path computation
    return U_i(s_I)
```

$\mathcal{G}$: SG, ε: precision requirement

Our algorithm for SGs

```
procedure BVI_WP(𝒢, ε)        𝒢: SG, ε: precision requirement
    L_0 ← ⊥, U_0 ← ⊤, i ← 0
    while U_i(s_I) − L_i(s_I) > ε do
        i++
        L_i ← 𝕏L_{i−1}                          // value iteration for lower bounds
        ℳ_i ← ℳ_PlRd(𝒢, L_i)                    // player reduction
        𝒲_i ← 𝒲_LcPg(ℳ_i, U_{i−1})              // local propagation
        U_i ← min{U_{i−1}, WPW(𝒲_i)}            // widest path computation
    return U_i(s_I)
```

<u>Theorem(P,T,H,H,2020).</u> Let the while loop iterate forever in the above algorithm. Then it generates a decreasing sequence of functions that converges to optimal reachability probability:

$$U_0 \geq U_1 \geq \cdots \geq U_i \xrightarrow{i \to \infty} V$$

Experimental result

model	Param.	#states	#trans	#EC	[K+, Ver.1] itr	[K+, Ver.1] time(s)	[K+, Ver.2] itr	[K+, Ver.2] time(s)	[K+, learning] itr	[K+, learning] visit%	[K+, learning] time(s)	**Our alg.** itr	**Our alg.** time(s)
mdsm	3	62245	151143	1	121	3	121	4	17339	49.3	15	120	5
	4	335211	882765	1	125	15	125	47	91301	42.1	86	124	38
cloud	5	8842	60437	4421	7	7	7	1	167	6.9	14	7	<1
	6	34954	274965	17477	11	177	11	5	41	0.6	3	11	1
	7	139402	1237525	69701	11	19721	11	62	41	0.2	4	11	5
teamform	3	12475	15228	2754	2	<1	2	<1	972	49.0	137	2	<1
	4	96665	116464	19800	2	<1	2	<1	4154	34.6	9603	2	<1
	5	907993	1084752	176760	2	<1	2	<1			TO	2	<1
investor	50	211321	673810	29690	441	184	441	249			TO	364	48
	100	807521	2587510	114390	801	3318		OOM			TO	688	736
manyECs	500	1004	3007	502	6	7	6	7			TO	5	<1
	1000	2004	6007	1002	6	51	6	51			TO	5	<1
	5000	10004	30007	5002		SO		SO			TO	5	<1

[K+] Kelmendi, E., Kramer, J., Kretnsky, J., Weininger, M.: *Value iteration for simple stochastic games: stopping criterion and learning algorithm*. Proc. CAV 2018

- Precision constant = 10^{-6}
  (i.e. an approx. with 10^{-6} error range is returned for each successful runs)
- Green shaded = fastest
- Gray shaded = computation failure
  (TO=timeout (6hours), OOM= out of memory, SO=stack overflow)

Experimental result

model	Param.	#states	#trans	#EC	[K+, Ver.1] itr	[K+, Ver.1] time(s)	[K+, Ver.2] itr	[K+, Ver.2] time(s)	[K+, learning] itr	[K+, learning] visit%	[K+, learning] time(s)	**Our alg.** itr	**Our alg.** time(s)
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cloud	5	8842	60437	4421	7	7	7	1	167	6.9	14	7	<1
	6	34954	274965	17477	11	177	11	5	41	0.6	3	11	1
	7	139402	1237525	69701	11	19721	11	62	41	0.2	4	11	5
teamform	3	12475	15228	2754	2	<1	2	<1	972	49.0	137	2	<1
	4	96665	116464	19800	2	<1	2	<1	4154	34.6	9603	2	<1
	5	907993	1084752	176760	2	<1	2	<1			TO	2	<1
investor	50	211321	673810	29690	441	184	441	249			TO	364	48
	100	807521	2587510	114390	801	3318		OOM			TO	688	736
manyECs	500	1004	3007	502	6	7	6	7			TO	5	<1
	1000	2004	6007	1002	6	51	6	51			TO	5	<1
	5000	10004	30007	5002		SO		SO			TO	5	<1

[K+] Kelmendi, E., Kramer, J., Kretnsky, J., Weininger, M.: *Value iteration for simple stochastic games: stopping criterion and learning algorithm*. Proc. CAV 2018

Fastest in 7/13 instances

- Precision constant = 10^{-6}
  (i.e. an approx. with 10^{-6} error range is returned for each successful runs)
- Green shaded = fastest
- Gray shaded = computation failure
  (TO=timeout (6hours), OOM= out of memory, SO=stack overflow)

Experimental result

model	Param.	#states	#trans	#EC	[K+, Ver.1] itr	[K+, Ver.1] time(s)	[K+, Ver.2] itr	[K+, Ver.2] time(s)	[K+, learning] itr	[K+, learning] visit%	[K+, learning] time(s)	**Our alg.** itr	**Our alg.** time(s)
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	7	139402	1237525	69701	11	19721	11	62	41	0.2	4	11	5
teamform	3	12475	15228	2754	2	<1	2	<1	972	49.0	137	2	<1
	4	96665	116464	19800	2	<1	2	<1	4154	34.6	9603	2	<1
	5	907993	1084752	176760	2	<1	2	<1			TO	2	<1
investor	50	211321	673810	29690	441	184	441	249			TO	364	48
	100	807521	2587510	114390	801	3318		OOM			TO	688	736
manyECs	500	1004	3007	502	6	7	6	7			TO	5	<1
	1000	2004	6007	1002	6	51	6	51			TO	5	<1
	5000	10004	30007	5002		SO		SO			TO	5	<1

[K+] Kelmendi, E., Kramer, J., Kretnsky, J., Weininger, M.: *Value iteration for simple stochastic games: stopping criterion and learning algorithm*. Proc. CAV 2018

Slow/failure sometimes

Stably fast

- Precision constant = 10^{-6}
  (i.e. an approx. with 10^{-6} error range is returned for each successful runs)
- Green shaded = fastest
- Gray shaded = computation failure
  (TO=timeout (6hours), OOM= out of memory, SO=stack overflow)

Summary

We introduced a novel algorithm of Bounded Value Iteration (BVI) which is faster than the existing one.

Future works

- Adapt the technique to a general reward setting (currently reachability only)
- Extend applicability of the technique to more complicated systems (e.g. black box ones)